The following are examples of problems that should help you prepare for the second midterm. Other places to find good problems are the review sections for chapters 5 and 6.

1. I will pick one or two integrals from the "hard integrals" already posted. There may be one or two other integration problems as well.

2. Suppose $f(x)$ satisfies $f'(x) > 0, f''(x) < 0$ for $0 \leq x \leq 5$. You are assigned to find numbers L and U such that
$$L \leq \int_0^5 f(x)\,dx \leq U$$
numerically.

   (a) How would you go about finding U and L? (I.e., which of the standard numerical integration techniques would you use and what are the formulas for these methods.)

   (b) What is the "worst case" for the difference between U and L for the techniques you chose in part (a).

3. Describe how you would go about deriving the formula for the arc length of the graph of $y = f(x)$ for $a \leq x \leq b$.

4. Describe how you would go about deriving the formula for the average value of a function $f(x)$ on $a \leq x \leq b$.

5. Find the general solution of
$$\frac{dy}{dx} = \frac{x^2}{y^3}.$$

6. Find the 4th degree Taylor polynomial for the solution of
$$\frac{dy}{dx} = 3y + \cos(x), \quad y(0) = 2.$$

7. In order to computer
$$\int_0^1 e^{x^2}\,dx$$
we could use a numerical technique OR approximate e^{x^2} with its Taylor polynomial. How many term of the Taylor polynomial would we need in order to get the same accuracy as using the Left Hand Rule with $\Delta x = 0.1$?