

561 Homework 1:

(1)

Sec 1.1 #2 Which of the following is linear?

a) $L[u] = u_x + x u_y$

This is linear: $L[u+v] = (u+v)_x + x(u+v)_y$

$$= u_x + v_x + x u_y + x v_y$$

$$= u_x + x u_y + v_x + x v_y = L[u] + L[v]$$

and $L[ku] = (ku)_x + x(ku)_y$

$$= k u_x + k x u_y = k L[u]$$

b) $L[u] = u_x + u u_y$

This is not linear

$$L[ku] = (ku)_x + (ku)(ku)_y = k u_x + k^2 u u_y$$

and

$$k L[u] = k u_x + k u u_y \text{ and these are not equal when } k \neq 1 \text{ or } 0$$

c) $L[u] = u_x + u_y^2$

This is not linear

$$L[ku] = (ku)_x + (ku)_y^2 = k u_x + k^2 u_y^2$$

while

$$k L[u] = k(u_x + u_y^2) = k u_x + k u_y^2$$

and these are not equal for $k \neq 1$ or 0 .

d) $L[u] = u_x + u_y + 1$

This is not linear

$$L[u+v] = (u+v)_x + (u+v)_y + 1 = u_x + v_x + u_y + v_y + 1$$

while

$$L[u] + L[v] = u_x + u_y + 1 + v_x + v_y + 1 = u_x + v_x + u_y + v_y + 2$$

and these are not equal.