3) Solve $(1+x^2)u_x + u_y = 0$.

We can write the equation as

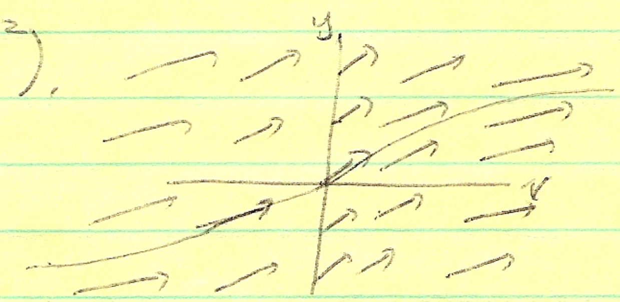
$$\begin{pmatrix} 1+x^2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} u_x \\ u_y \end{pmatrix} = 0$$

or $$\nabla u \cdot \begin{pmatrix} 1+x^2 \\ 1 \end{pmatrix} = 0$$

That is, at (x,y), the directional derivative of u in the direction $\begin{pmatrix} 1+x^2 \\ 1 \end{pmatrix}$ is 0.

This defines a vector field on $\mathbb{R}^2$; Let

$$F\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1+x^2 \\ 1 \end{pmatrix}.$$



and u must be constant along curves that are tangent to this direction field, i.e.

u is constant along solutions of $\begin{cases} \dfrac{dx}{dt} = 1+x^2 \\ \dfrac{dy}{dt} = 1 \end{cases}$

To write these solution curves as graphs of y as a function of x we need the slope of solution curves at each point i.e. $\dfrac{dy}{dx} = \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{dy/dt}{dx/dt}$