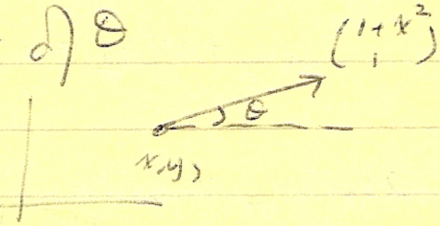
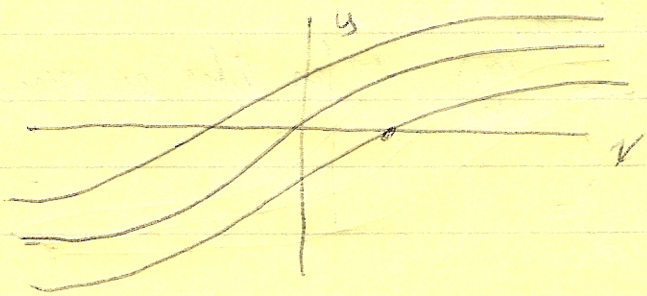
(note $\frac{dy/dt}{dx/dt}$ is the tangent of θ in the picture to right)



So $\frac{dy}{dx} = \frac{1}{1+x^2}$ or $y(x) = \arctan(x) + c$

So the solution $u(x,y)$ of $(1+x^2)u_x + u_y = 0$

is constant along characteristic curves $y(x) = \arctan(x) + c$



$y - \arctan(x) = c$

i.e. $u(x,y) = f(y - \arctan(x))$

for any function $f: \mathbb{R} \to \mathbb{R}$

Check $(1+x^2)u_x + u_y = (1+x^2) f'(y - \arctan(x)) \frac{d(\arctan(x))}{dx}$

$+ f'(y - \arctan(x))$

$= 0$ ✓.