7.) Solve $a u_x + b u_y + cu = 0$.

What to do? -- Must try something -

Rewrite

$$a u_x + b u_y = -cu,$$

or

$$\nabla u \cdot \binom{a}{b} = -cu \quad \text{i.e. in direction } \binom{a}{b},$$

u looks like exponential decay... if this exponential decay were just in some simple direction -

So lets do a change of coordinates

Let

$$\tilde{x} = \tilde{x}(x,y) = \alpha x + \beta y$$
$$\tilde{y} = \tilde{y}(x,y) = \gamma x + \delta y.$$

and

$$\tilde{u}(\tilde{x},\tilde{y}) = \tilde{u}(\tilde{x}(x,y), \tilde{y}(x,y)) = u(x,y)$$

Then

$$u_x = \tilde{u}_{\tilde{x}} \frac{\partial \tilde{x}}{\partial x} + \tilde{u}_{\tilde{y}} \frac{\partial \tilde{y}}{\partial x} = \tilde{u}_{\tilde{x}} \alpha + \tilde{u}_{\tilde{y}} \gamma$$

$$u_y = \tilde{u}_{\tilde{x}} \frac{\partial \tilde{x}}{\partial y} + \tilde{u}_{\tilde{y}} \frac{\partial \tilde{y}}{\partial y} = \tilde{u}_{\tilde{x}} \beta + \tilde{u}_{\tilde{y}} \delta.$$

If $a u_x + b u_y + cu = 0$ then

$$a(\alpha \tilde{u}_{\tilde{x}} + \gamma \tilde{u}_{\tilde{y}}) + b(\beta \tilde{u}_{\tilde{x}} + \delta \tilde{u}_{\tilde{y}}) + c\tilde{u} = 0$$

$$(a\alpha + b\beta)\tilde{u}_{\tilde{x}} + (a\gamma + b\delta)\tilde{u}_{\tilde{y}} + c\tilde{u} = 0$$