11. Use the coordinate method to solve
$$u_x + 2u_y + (2x-y)u = 2x^2+3xy-2y^2.$$

Well we could just substitute $\begin{cases} \tilde{x} = \alpha x + \beta y \\ \tilde{y} = \gamma x + \delta y \end{cases}$

but to substitute on the right we'd need x and y as functions of $\tilde{x}, \tilde{y}$ -- not so nice ...
So let's think first.

Note $2x^2+3xy-2y^2 = (2x-y)\cdot(x+2y)$.

and note the $2x-y$ appears on the left. -- that's enough to give the following a try.

$$\begin{cases} \tilde{x} = 2x-y \\ \tilde{y} = x+2y \end{cases}$$

$u_x = 2\tilde{u}_{\tilde{x}} + \tilde{u}_{\tilde{y}}$ and $u_y = -\tilde{u}_{\tilde{x}} + 2\tilde{u}_{\tilde{y}}$
So the equation becomes.
$$2\tilde{u}_{\tilde{x}} + \tilde{u}_{\tilde{y}} + 2(-\tilde{u}_{\tilde{x}} + 2\tilde{u}_{\tilde{y}}) + \tilde{x}\tilde{u} = \tilde{x}\cdot\tilde{y}$$

I'm told this should be a 5!

$$3\tilde{u}_{\tilde{y}} + \tilde{x}\tilde{u} = \tilde{x}\,\tilde{y}.$$

or

$$\tilde{u}_{\tilde{y}} = -\frac{\tilde{x}}{3}\tilde{u} + \frac{\tilde{x}\tilde{y}}{3}.$$

Thinking of $\tilde{x}$ as constant this is a linear ode in $\tilde{y}$

$$\frac{\partial\tilde{u}}{\partial\tilde{y}} = -k\tilde{u} + k\tilde{y} \qquad (\text{where } k = \tilde{x}/3).$$