

Solving the ode. (Never forget anything).

$$\bar{u} = ce^{-ky} + \bar{y} - 1/k.$$

or

$$\tilde{u}(\tilde{x}, \tilde{y}) = f(\tilde{x}) e^{-\frac{\tilde{x}}{3}\tilde{y}} + \tilde{y} - 1/3. \text{ for any function } f.$$

or

$$u(x, y) = f(2x-y) e^{-\frac{(2x-y)(x+2y)}{3}} + (x+2y) - \frac{3}{2x-y}.$$