

$$e) L[u] = \sqrt{1+x^2} \cos y u_x + u_{yxy} - [\arctan(\frac{x}{y})] u.$$

This is linear

$$\begin{aligned} L[u+v] &= \sqrt{1+x^2} \cos y (u+v)_x + (u+v)_{yxy} - \arctan(\frac{x}{y})(u+v) \\ &= \sqrt{1+x^2} \cos y (u_x) + \sqrt{1+x^2} \cos y v_x \\ &\quad + u_{yxy} + v_{yxy} + \arctan(\frac{x}{y}) u \\ &\quad + \arctan(\frac{x}{y}) v \\ &= L[u] + L[v] \end{aligned}$$

$$\begin{aligned} \text{and } L[ku] &= \sqrt{1+x^2} \cos y (ku)_x + (ku)_{yxy} - \arctan(\frac{x}{y})(ku) \\ &= k \sqrt{1+x^2} \cos y u_x + k u_{yxy} - k \arctan(\frac{x}{y}) u \\ &= k L[u], \end{aligned}$$

- 3) a) This is 2nd order, linear, non-homogeneous (the "+1")
 b) This is 2nd order, linear, homogeneous
 c) This is 3rd order, non-linear (the uu_x term), homogeneous
 d) This is 2nd order, linear, non-homogeneous (the " x^2 ")
 e) This is 2nd order, linear, homogeneous.
 f) This is 1st order, non-linear ($\sqrt{1+u_x^2}$), homogeneous
 g) This is 1st order, linear, homogeneous
 h) This is 4th order, non-linear ($\sqrt{1+u}$ term), homogeneous.