

7) To check if $1+x$, $1-x$ and $1+x+x^2$, we ask if we can find constants a, b, c , not all zero such

that $a(1+x) + b(1-x) + c(1+x+x^2) = 0$ (for all x)
i.e.

$$a + ax + b - bx + c + cx + cx^2 = 0$$

or

$$(a+b+c) + (a-b+c)x + cx^2 = 0.$$

Well, to get 0 for all x we need $c=0$.

So we consider $(a+b) + (a-b)x = 0$.

If $x=0$ then $a+b=0$

If $x=1$ then $a+b+(a-b)=0$

so $2a=0$ so $a=0$

hence $a+b=0 \Rightarrow b=0$

So the only choice of a, b, c that gives 0 for all x is $a=b=c=0$. So they are independent functions