10) Consider $\frac{d^3u}{dx^3} - 3\frac{d^2u}{dx^2} + 4u = 0$.

Let $V = \{u(x) : u \text{ is a solution}\}$.

Note $u(x) \equiv 0$ is a solution so $V \neq \emptyset$.

If $u \in V$ then $\frac{d^3u}{dx^3} - 3\frac{d^2u}{dx^2} + 4u = 0$

So for any constant k $\frac{d^3(ku)}{dx^3} - 3\frac{d^2(ku)}{dx^2} + 4ku = 0$

so $ku \in V$.

Also if $u, v \in V$ then $\frac{d^3u}{dx^3} - 3\frac{d^2u}{dx^2} + 4u = 0$

and $\frac{d^3v}{dx^3} - 3\frac{d^2v}{dx^2} + 4v = 0$

So $\frac{d^3(u+v)}{dx^3} - 3\frac{d^2(u+v)}{dx^2} + 4(u+v) = 0$

or $u+v \in V$

So V is a vector space.

Now for a trip down memory lane —

Make this a 1st order system

$$\begin{cases} \frac{du}{dx} = v \\ \frac{dv}{dx} = \frac{d^2u}{dx^2} = w \\ \frac{dw}{dx} = \frac{d^3u}{dx^3} = 3\frac{d^2u}{dx^2} - 4u = 3w - 4u \end{cases}$$

Let $\bar{U} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$ so $\frac{d\bar{U}}{dx} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 0 & 3 \end{bmatrix} \bar{U}$.