So we guess $\vec{U}(x) = k_2 e^{2x}\begin{pmatrix}4\\2\\1\end{pmatrix}$ is a solution of the system or

$u(x) = k_2 e^{2x}$ is a solution of the equation (Check again ...)

Since 2 is a double root, we try the traditional guess $u(x) = k_3 x e^{2x}$ as the last soln.

$$u' = k_3 e^{2x} + 2k_3 x e^{2x}$$
$$u'' = 2k_3 e^{2x} + 2k_3 e^{2x} + 4k_3 x e^{2x} = 4k_3 e^{2x} + 4k_3 x e^{2x}$$
$$u''' = 8k_3 e^{2x} + 4k_3 e^{2x} + 8k_3 x e^{2x} = 12k_3 e^{2x} + 8k_3 x e^{2x}$$

and $u''' - 3u'' + 4u = 0$ so the general solution is

$$u(x) = k_1 e^{-x} + k_2 e^{2x} + k_3 x e^{2x}$$

The dimension of the space of solutions is 3 and a basis is e^{-x}, e^{2x}, xe^{2x}.

11. Verify $u(x,y) = f(x)g(y)$ is a soln of $u u_{xy} = u_x u_y$.

Check $u_x = f'(x)g(y)$, $u_y = f(x)g'(y)$

$u_{xy} = f'(x)g'(y)$ so

$u u_{xy} = (f(x)g(y))(f'(x)g'(y))$

and

$u_x u_y = f'(x)g(y)\, f(x)g'(y)$. These are always equal so $u(x,y) = f(x)g(y)$ is a solution.