12) Verify $u_n(x,y) = \sin nx \sinh ny$
is a solution of $u_{xx} + u_{yy} = 0$ for all $n > 0$.

Recall $\sinh(ny) = \dfrac{e^{ny} - e^{-ny}}{2}$.

More important $\dfrac{d}{dz}\sinh(z) = \cosh(z)$
and $\dfrac{d}{dz}\cosh(z) = \sinh(z)$

So $\dfrac{d^2 \sinh(ny)}{dy^2} = n^2 \sinh(y)$

So $\dfrac{\partial^2 u_n}{\partial x^2} = -n^2 \sin nx \sinh ny$

$\dfrac{\partial^2 u_n}{\partial y^2} = n^2 \sin nx \sinh ny$

and $u_{xx} + u_{yy} = 0$.

1.2 #1 Solve $2u_t + 3u_x = 0$ with $u(0,x) = \sin x$.

We seek solutions in the form $u(t,x) = f(at + bx)$
Plugging in, we have

$$2u_t + 3u_x = 2a f'(at+bx) + 3b f'(at+bx)$$
$$= (2a+3b) f'(at+bx)$$

So we need only choose a and b so that
$2a + 3b = 0$ or $a = -3, b = 2$.
i.e. the characteristics are $-3t + 2x = \text{constant}$.
or $-\frac{3}{2}t + x = \text{constant}$
or $-3t = -2x + k$ or $t = \frac{2}{3}x + k$
k any constant.