2. Solve $3u_y + u_{xy} = 0$.

Hint: Let $v = u_y$.

Taking the hint, we get $3v + v_x = 0$.

For each fixed y this is just the ode.

$$v_x = -3v$$

so

$$v(x,y) = k\,e^{-3x}$$ where the constant k can depend on y, i.e.

$$v(x,y) = k(y)\,e^{-3x}$$

So $u_y(x,y) = k(y)\,e^{-3x}$.

Constant term depends on x

or $\int u_y(x,y)\,dy = \int k(y)\,e^{-3x}\,dy = e^{-3x}\int k(y)\,dy$.

and we get $u(x,y) = K(y)\,e^{-3x} + g(x)$ (K any anti-derivative of k).

i.e. The general solution is $u(x,y) = f(y)\,e^{-3x} + g(x)$ for any $f(y)$ and $g(x)$

Checking $u_y = f'(y)\,e^{-3x}$, $u_{xy} = -3f'(y)\,e^{-3x}$

so

$3u_y + u_{xy} = 0$, it checks.