From Calculus
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***7.** Potpourri. (No holds barred.) The following integrations involve all the methods of the previous problems

(i) $\displaystyle\int \frac{\arctan x}{1+x^2}\,dx.$

(ii) $\displaystyle\int \frac{x\arctan x}{(1+x^2)^3}\,dx.$

(iii) $\displaystyle\int \log\sqrt{1+x^2}\,dx.$

(iv) $\displaystyle\int x\log\sqrt{1+x^2}\,dx.$

(v) $\displaystyle\int \frac{x^2-1}{x^2+1}\cdot\frac{1}{\sqrt{1+x^4}}\,dx.$

(vi) $\displaystyle\int \arcsin\sqrt{x}\,dx.$

(vii) $\displaystyle\int \frac{x}{1+\sin x}\,dx.$

(viii) $\displaystyle\int e^{\sin x}\cdot\frac{x\cos^3 x-\sin x}{\cos^2 x}\,dx.$

(ix) $\displaystyle\int \sqrt{\tan x}\,dx.$

(x) $\displaystyle\int \frac{dx}{x^6+1}.$ (To factor $x^6+1$, first factor $y^3+1$, using Problem 1-1.)

The following two problems provide still more practice at integration, if you need it (and can bear it). Problem 8 involves algebraic and trigonometric manipulations and integration by parts, while Problem 9 involves substitutions. (Of course, in many cases the resulting integrals will require still further manipulations.)

**8.** Find the following integrals.

(i) $\displaystyle\int \log(a^2+x^2)\,dx.$

(ii) $\displaystyle\int \frac{1+\cos x}{\sin^2 x}\,dx.$

(iii) $\displaystyle\int \frac{x+1}{\sqrt{4-x^2}}\,dx.$

(iv) $\displaystyle\int x\arctan x\,dx.$

(v) $\displaystyle\int \sin^3 x\,dx.$

(vi) $\displaystyle\int \frac{\sin^3 x}{\cos^2 x}\,dx.$

(vii) $\displaystyle\int x^2 \arctan x \, dx.$

(viii) $\displaystyle\int \frac{x\,dx}{\sqrt{x^2 - 2x + 2}}.$

(ix) $\displaystyle\int \sec^3 x \tan x \, dx.$

(x) $\displaystyle\int x \tan^2 x \, dx.$

**9.** Find the following integrals.

(i) $\displaystyle\int \frac{dx}{(a^2 + x^2)^2}.$

(ii) $\displaystyle\int \sqrt{1 - \sin x} \, dx.$

(iii) $\displaystyle\int \arctan \sqrt{x} \, dx.$

(iv) $\displaystyle\int \sin \sqrt{x + 1} \, dx.$

(v) $\displaystyle\int \frac{\sqrt{x^3 - 2}}{x} \, dx.$

(vi) $\displaystyle\int \log(x + \sqrt{x^2 - 1}\,) \, dx.$

(vii) $\displaystyle\int \log(x + \sqrt{x}\,) \, dx.$

(viii) $\displaystyle\int \frac{dx}{x - x^{3/5}}.$

(ix) $\displaystyle\int (\arcsin x)^2 \, dx.$

(x) $\displaystyle\int x^5 \arctan(x^2) \, dx.$

**10.** If you have done Problem 18-9, the integrals (ii) and (iii) in Problem 4 will lo very familiar. In general, the substitution $x = \cosh u$ often works for integr involving $\sqrt{x^2 - 1}$, while $x = \sinh u$ is the thing to try for integrals involvi $\sqrt{x^2 + 1}$. Try these substitutions on the other integrals in Problem 4. (T method is not really recommended; it is easier to stick with trigonomet substitutions.)

***11.** The world's sneakiest substitution is undoubtedly

$$t = \tan \frac{x}{2}, \qquad x = 2 \arctan t,$$
$$dx = \frac{2}{1 + t^2} \, dt.$$