

SERGE LANG

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Serge Lang was a man of exceptional generosity. If you happened to be taking one of his afternoon courses then after almost every class you were treated to a snack at a local café, where your mathematical education continued in a more informal setting. If you were one of Serge's research students then he would shower you with complimentary copies of his books and do everything in his power to advance your career. If you were not one of his students but simply a young mathematician whose research had excited his interest then he would likewise champion your efforts and do what he could to help you. On occasion Serge's generosity appears to have taken the form of direct financial assistance to impecunious members of the mathematical community. These subventions were discreet and their scope is unknown to me.

Serge was also a man of intense interests and strong convictions, and his occasionally unconventional reactions to conventional situations were in part an uncensored expression of his intensity. When I went to his office one day to request a thesis problem, his immediate response was something like this: "Well, of course there are always hopeless problems, for example – [a problem is mentioned in a flash of lightning] – I don't recommend that you work on that." I suspect that the problem in question was something that he had been thinking about very hard, and apparently he felt that he had not gained any insight that could be passed on to a graduate student. But such was the hold of the problem on his imagination that he could not help alluding to it anyway. Uttered by anybody else, his words to me might have sounded discouraging, but coming from Serge their effect was if anything inspiring. I did not leave his office with a thesis problem that day, but perhaps I left in greater awe of mathematics itself.

The problem, by the way, was not something that I grasped at the time, but in retrospect it amounted to this: Which smooth projective curves defined over $\overline{\mathbb{Q}}$ arise as quotients of the upper half-plane by arithmetic subgroups of $SL(2, \mathbb{Q})$? The conversation occurred around 1974, and five years later Belyi published his remarkable discovery: They all do. The episode illustrates Lang's knack for asking prescient questions, but it also represents an atypical failure to make the appropriate conjecture. More emblematic of Lang's career are the many conjectures – in particular those pertaining to diophantine properties of varieties over number fields – which stimulated research and received at least partial validation in his lifetime. For example there is the conjectured lower bound for heights of nontorsion points on elliptic curves (proved for elliptic curves with integral j -invariant by Silverman in 1981, and reduced to Szpiro's conjecture by Silverman and Hindry in 1988), or the conjecture that if a subvariety of an abelian variety contains infinitely many rational points then it contains a translate of an abelian subvariety of dimension > 0 (proved by Faltings in 1990), or the conjecture that the set of rational points on

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$ - $\mathcal{T}\mathcal{E}\mathcal{X}$

a variety of general type is not Zariski-dense (explored from a geometric as well as an arithmetic standpoint in work of Caporaso-Harris-Mazur, Abramovich, Pacelli, Abramovich-Voloch, Hassett, and others). As Lang was well aware, this last conjecture has a striking application to \mathcal{M}_g , the moduli space of curves of genus g : Since \mathcal{M}_g is of general type for large g (Harris-Mumford), it would follow that certain algebraic identities are forced on a curve of genus g simply by virtue of its being defined over a given number field.

The conjectures just mentioned pertain to refinements of Siegel's theorem on the finiteness of integral points on curves of genus ≥ 1 and generalizations of Mordell's conjecture (now Faltings's theorem) on the finiteness of rational points on curves of genus ≥ 2 . But as dear as such matters were to Lang's heart, and as prominently as they must figure in an appraisal of his legacy, the fact remains that many of Lang's best-known results and conjectures lie outside the domain of diophantine geometry. Consider for instance the theorem on the triviality of principal homogeneous spaces over finite fields, or the formulas for orders of cuspidal divisor class groups on modular curves (joint work with Kubert), or the conjectures on Frobenius distributions in $GL(2)$ extensions (joint work with Trotter). There are also substantial portions of Lang's work, such as the forays into Nevanlinna theory or into complex hyperbolic geometry, which lie outside of number theory altogether, even if the motivation is number-theoretic. In any case, however broad his interests as a research mathematician, Lang's interests as a mathematical educator were broader: His graduate and undergraduate textbooks and his talks for high school students and the general public span much of mathematics, making him a "one-man Bourbaki," in the words of a German reviewer of one of his books. There are not many figures in mathematics to whom this epithet could be applied.

I last saw Serge on May 2, 2005, when he gave a talk in the algebra seminar at Boston University. He was his usual lively self, excited not only about his lecture topic ("The error term in the *abc* conjecture and diophantine approximation") but also about what he saw as an impending reorientation of algebraic geometry toward closer connections with analysis and toward a reduction of general theories to fundamental special cases. In part the issue that concerned him seemed to be pedagogical, for he stressed the need to rewrite the textbooks in the field, adding in the same breath that he could not be the one to do it. But he had more than pedagogy in mind, and given his prescience I do not doubt that he was on to something, even if he was unable to formulate it precisely. Perhaps subsequent developments in mathematics will confirm the soundness of his instincts. Be that as it may, his seminar talk was well received, and as so often in the past, the name Serge Lang had drawn a good crowd. After the seminar dinner we adjourned to my house for coffee and ice cream, but I don't recall that Serge had either: It seemed that he was standing by the piano the whole evening, engaged in animated conversation with one person after another, or with several at once.

There is a lot more to say about Serge, but at some point the reminiscences of those who knew him may be less evocative than the lyrics of a songwriter who didn't. Serge loved a variety of genres of music, and at Yale he even performed as the lutenist in a recital of Elizabethan songs. Here the final word will be left not to John Dowland but rather to another of Serge's favorites, the folksinger Phil Ochs, whose death in 1976 affected him deeply. If you never knew Serge, then listen to the song *When I'm gone*. It captures some of his spirit.