2.8

5. The derivative f' is increasing when the slopes of the tangent lines of f are becoming larger as x increases. This seems to be the case on the interval $(2, 5)$. The derivative is decreasing when the slopes of the tangent lines of f are becoming smaller as x increases, and this seems to be the case on $(-\infty, 2)$ and $(5, \infty)$. So f' is increasing on $(2, 5)$ and decreasing on $(-\infty, 2)$ and $(5, \infty)$.

10. (a) The rate of increase of the population is initially very small, then gets larger until it reaches a maximum at about $t = 8$ hours, and decreases toward 0 as the population begins to level off.

(b) The rate of increase has its maximum value at $t = 8$ hours.

(c) The population function is concave upward on $(0, 8)$ and concave downward on $(8, 18)$.

(d) At $t = 8$, the population is about 350, so the inflection point is about $(8, 350)$.

12. (a) If the position function is increasing, then the particle is moving toward the right. This occurs on t-intervals $(0, 2)$ and $(4, 6)$. If the function is decreasing, then the particle is moving toward the left—that is, on $(2, 4)$.

(b) The acceleration is the second derivative and is positive where the curve is concave upward. This occurs on $(3, 6)$. The acceleration is negative where the curve is concave downward—that is, on $(0, 3)$.

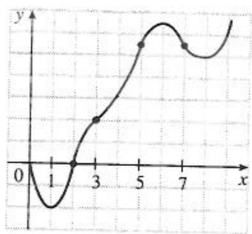
16. (a) f is increasing where f' is positive, on $(1, 6)$ and $(8, \infty)$, and decreasing where f' is negative, on $(0, 1)$ and $(6, 8)$.

(b) f has a local maximum where f' changes from positive to negative, at $x = 6$, and local minima where f' changes from negative to positive, at $x = 1$ and at $x = 8$.

(c) f is concave upward where f' is increasing, that is, on $(0, 2)$, $(3, 5)$, and $(7, \infty)$, and concave downward where f' is decreasing, that is, on $(2, 3)$ and $(5, 7)$.

(d) There are points of inflection where f changes its direction of concavity, at $x = 2$, $x = 3$, $x = 5$ and $x = 7$.

(e)



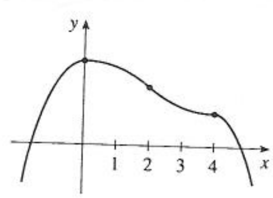
19. $f'(0) = f'(4) = 0 \Rightarrow$ horizontal tangents at $x = 0, 4$.

$f'(x) > 0$ if $x < 0 \Rightarrow f$ is increasing on $(-\infty, 0)$.

$f'(x) < 0$ if $0 < x < 4$ or if $x > 4 \Rightarrow f$ is decreasing on $(0, 4)$ and $(4, \infty)$.

$f''(x) > 0$ if $2 < x < 4 \Rightarrow f$ is concave upward on $(2, 4)$.

$f''(x) < 0$ if $x < 2$ or $x > 4 \Rightarrow f$ is concave downward on $(-\infty, 2)$ and $(4, \infty)$. There are inflection points when $x = 2$ and 4.



28. (a) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{[(x+h)^4 - 2(x+h)^2] - (x^4 - 2x^2)}{h}$

$$= \lim_{h\to 0} \frac{(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - 2x^2 - 4xh - 2h^2) - (x^4 - 2x^2)}{h}$$

$$= \lim_{h\to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4 - 4xh - 2h^2}{h} = \lim_{h\to 0} (4x^3 + 6x^2h + 4xh^2 + h^3 - 4x - 2h) = 4x^3 - 4x$$

$$f''(x) = \lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h} = \lim_{h\to 0} \frac{[4(x+h)^3 - 4(x+h)] - (4x^3 - 4x)}{h}$$

$$= \lim_{h\to 0} \frac{(4x^3 + 12x^2h + 12xh^2 + 4h^3 - 4x - 4h) - (4x^3 - 4x)}{h} = \lim_{h\to 0} \frac{12x^2h + 12xh^2 + 4h^3 - 4h}{h}$$

$$= \lim_{h\to 0} (12x^2 + 12xh + 4h^2 - 4) = 12x^2 - 4$$

(b) $f'(x) > 0 \;\Leftrightarrow\; 4x^3 - 4x > 0 \;\Leftrightarrow\; 4x(x^2-1) > 0 \;\Leftrightarrow\; 4x(x+1)(x-1) > 0$, so f is increasing on $(-1, 0)$ and $(1, \infty)$ and f is decreasing on $(-\infty, -1)$ and $(0, 1)$.

(c) $f''(x) > 0 \;\Leftrightarrow\; 12x^2 - 4 > 0 \;\Leftrightarrow\; 12x^2 > 4 \;\Leftrightarrow\; x^2 > \frac{1}{3} \;\Leftrightarrow\; |x| > \sqrt{\frac{1}{3}}$, so f is CU on $\left(-\infty, -\sqrt{\frac{1}{3}}\right)$ and $\left(\sqrt{\frac{1}{3}}, \infty\right)$ and f is CD on $\left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right)$.

3.1

15. $y = 3e^x + \dfrac{4}{\sqrt[3]{x}} = 3e^x + 4x^{-1/3} \quad \Rightarrow \quad y' = 3(e^x) + 4(-\frac{1}{3})x^{-4/3} = 3e^x - \frac{4}{3}x^{-4/3}$

25. $z = \dfrac{A}{y^{10}} + Be^y = Ay^{-10} + Be^y \quad \Rightarrow \quad z' = -10Ay^{-11} + Be^y = -\dfrac{10A}{y^{11}} + Be^y$

3.2

3. By the Product Rule, $f(x) = (x^3 + 2x)e^x \;\; \Rightarrow$

$$f'(x) = (x^3 + 2x)(e^x)' + e^x(x^3 + 2x)' = (x^3 + 2x)e^x + e^x(3x^2 + 2)$$
$$= e^x[(x^3 + 2x) + (3x^2 + 2)] = e^x(x^3 + 3x^2 + 2x + 2)$$

6. By the Quotient Rule, $y = \dfrac{e^x}{1+x} \;\; \Rightarrow \;\; y' = \dfrac{(1+x)\,e^x - e^x(1)}{(1+x)^2} = \dfrac{e^x + xe^x - e^x}{(x+1)^2} = \dfrac{xe^x}{(x+1)^2}.$

The notations $\overset{\text{PR}}{\Rightarrow}$ and $\overset{\text{QR}}{\Rightarrow}$ indicate the use of the Product and Quotient Rules, respectively.

12. $y = \dfrac{x+1}{x^3 + x - 2} \;\; \overset{\text{QR}}{\Rightarrow}$

$$y' = \frac{(x^3 + x - 2)(1) - (x+1)(3x^2 + 1)}{(x^3 + x - 2)^2} = \frac{x^3 + x - 2 - 3x^3 - 3x^2 - x - 1}{(x^3 + x - 2)^2} = \frac{-2x^3 - 3x^2 - 3}{(x^3 + x - 2)^2}$$

or $-\dfrac{2x^3 + 3x^2 + 3}{(x-1)^2(x^2 + x + 2)^2}$

19. $f(t) = \dfrac{2t}{2 + \sqrt{t}} \;\; \overset{\text{QR}}{\Rightarrow} \;\; f'(t) = \dfrac{(2 + t^{1/2})(2) - 2t\left(\frac{1}{2}t^{-1/2}\right)}{(2 + \sqrt{t})^2} = \dfrac{4 + 2t^{1/2} - t^{1/2}}{(2 + \sqrt{t})^2} = \dfrac{4 + t^{1/2}}{(2 + \sqrt{t})^2}$ or $\dfrac{4 + \sqrt{t}}{(2 + \sqrt{t})^2}$

27. $f(x) = \dfrac{x^2}{1 + 2x} \;\; \Rightarrow \;\; f'(x) = \dfrac{(1 + 2x)(2x) - x^2(2)}{(1 + 2x)^2} = \dfrac{2x + 4x^2 - 2x^2}{(1 + 2x)^2} = \dfrac{2x^2 + 2x}{(1 + 2x)^2} \;\; \Rightarrow$

$$f''(x) = \frac{(1 + 2x)^2(4x + 2) - (2x^2 + 2x)(1 + 4x + 4x^2)'}{[(1 + 2x)^2]^2} = \frac{2(1 + 2x)^2(2x + 1) - 2x(x + 1)(4 + 8x)}{(1 + 2x)^4}$$
$$= \frac{2(1 + 2x)[(1 + 2x)^2 - 4x(x + 1)]}{(1 + 2x)^4} = \frac{2(1 + 4x + 4x^2 - 4x^2 - 4x)}{(1 + 2x)^3} = \frac{2}{(1 + 2x)^3}$$

3.3

2. $y = 2\csc x + 5\cos x \quad \Rightarrow \quad y' = -2\csc x\,\cot x - 5\sin x$

5. $y = \sec\theta\,\tan\theta \quad \Rightarrow \quad y' = \sec\theta\,(\sec^2\theta) + \tan\theta\,(\sec\theta\tan\theta) = \sec\theta\,(\sec^2\theta + \tan^2\theta)$. Using the identity $1 + \tan^2\theta = \sec^2\theta$, we can write alternative forms of the answer as $\sec\theta\,(1 + 2\tan^2\theta)$ or $\sec\theta\,(2\sec^2\theta - 1)$.

11. $f(\theta) = \dfrac{\sec\theta}{1+\sec\theta} \quad \Rightarrow$

$$f'(\theta) = \frac{(1+\sec\theta)(\sec\theta\,\tan\theta) - (\sec\theta)(\sec\theta\,\tan\theta)}{(1+\sec\theta)^2} = \frac{(\sec\theta\,\tan\theta)\,[(1+\sec\theta) - \sec\theta]}{(1+\sec\theta)^2} = \frac{\sec\theta\,\tan\theta}{(1+\sec\theta)^2}$$

16. $\dfrac{d}{dx}(\sec x) = \dfrac{d}{dx}\left(\dfrac{1}{\cos x}\right) = \dfrac{(\cos x)(0) - 1(-\sin x)}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x} = \dfrac{1}{\cos x}\cdot\dfrac{\sin x}{\cos x} = \sec x\,\tan x$