3-4

6. Let $u = g(x) = 2 - e^x$ and $y = f(u) = \sqrt{u}$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = \left(\frac{1}{2}u^{-1/2}\right)(-e^x) = -\dfrac{e^x}{2\sqrt{2 - e^x}}$.

9. $F(x) = \sqrt{1 - 2x} = (1 - 2x)^{1/2} \quad \Rightarrow \quad F'(x) = \frac{1}{2}(1 - 2x)^{-1/2}(-2) = -\dfrac{1}{\sqrt{1 - 2x}}$

11. $f(z) = \dfrac{1}{z^2 + 1} = (z^2 + 1)^{-1} \quad \Rightarrow \quad f'(z) = -1(z^2 + 1)^{-2}(2z) = -\dfrac{2z}{(z^2 + 1)^2}$

25. $y = \sec^2 x + \tan^2 x = (\sec x)^2 + (\tan x)^2 \quad \Rightarrow$

$y' = 2(\sec x)(\sec x\,\tan x) + 2(\tan x)(\sec^2 x) = 2\sec^2 x\,\tan x + 2\sec^2 x\,\tan x = 4\sec^2 x\,\tan x$

26. $y = \dfrac{e^u - e^{-u}}{e^u + e^{-u}} \quad \Rightarrow$

$$y' = \frac{(e^u + e^{-u})(e^u - (-e^u)) - (e^u - e^{-u})(e^u + (-e^{-u}))}{(e^u + e^{-u})^2} = \frac{e^{2u} + e^0 + e^0 + e^{-2u} - (e^{2u} - e^0 - e^0 + e^{-2u})}{(e^u + e^{-u})^2}$$

$$= \frac{4e^0}{(e^u + e^{-u})^2} = \frac{4}{(e^u + e^{-u})^2}$$

39. $y = e^{\alpha x}\sin\beta x \quad \Rightarrow \quad y' = e^{\alpha x}\cdot\beta\cos\beta x + \sin\beta x\cdot\alpha e^{\alpha x} = e^{\alpha x}(\beta\cos\beta x + \alpha\sin\beta x) \quad \Rightarrow$

$$y'' = e^{\alpha x}(-\beta^2\sin\beta x + \alpha\beta\cos\beta x) + (\beta\cos\beta x + \alpha\sin\beta x)\cdot\alpha e^{\alpha x}$$

$$= e^{\alpha x}(-\beta^2\sin\beta x + \alpha\beta\cos\beta x + \alpha\beta\cos\beta x + \alpha^2\sin\beta x) = e^{\alpha x}(\alpha^2\sin\beta x - \beta^2\sin\beta x + 2\alpha\beta\cos\beta x)$$

$$= e^{\alpha x}\left[(\alpha^2 - \beta^2)\sin\beta x + 2\alpha\beta\cos\beta x\right]$$

3.5

3. $\dfrac{d}{dx}\left(x^3+y^3\right)=\dfrac{d}{dx}(1) \;\Rightarrow\; 3x^2+3y^2\cdot y'=0 \;\Rightarrow\; 3y^2\,y'=-3x^2 \;\Rightarrow\; y'=-\dfrac{x^2}{y^2}$

8. $\dfrac{d}{dx}\left(y^5+x^2y^3\right)=\dfrac{d}{dx}\left(1+ye^{x^2}\right) \;\Rightarrow\; 5y^4\,y'+\left(x^2\cdot 3y^2\,y'+y^3\cdot 2x\right)=0+y\cdot e^{x^2}\cdot 2x+e^{x^2}\cdot y' \;\Rightarrow$

$y'\left(5y^4+3x^2y^2-e^{x^2}\right)=2xye^{x^2}-2xy^3 \;\Rightarrow\; y'=\dfrac{2xy\left(e^{x^2}-y^2\right)}{5y^4+3x^2y^2-e^{x^2}}$

9. $\dfrac{d}{dx}\left(x^2y^2+x\sin y\right)=\dfrac{d}{dx}(4) \;\Rightarrow\; x^2\cdot 2y\,y'+y^2\cdot 2x+x\cos y\cdot y'+\sin y\cdot 1=0 \;\Rightarrow$

$2x^2y\,y'+x\cos y\cdot y'=-2xy^2-\sin y \;\Rightarrow\; (2x^2y+x\cos y)y'=-2xy^2-\sin y \;\Rightarrow\; y'=\dfrac{-2xy^2-\sin y}{2x^2y+x\cos y}$

12. $\dfrac{d}{dx}\left[y\sin(x^2)\right]=\dfrac{d}{dx}\left[x\sin(y^2)\right] \;\Rightarrow\; y\cos(x^2)\cdot 2x+\sin(x^2)\cdot y'=x\cos(y^2)\cdot 2y\,y'+\sin(y^2)\cdot 1 \;\Rightarrow$

$y'\left[\sin(x^2)-2xy\cos(y^2)\right]=\sin(y^2)-2xy\cos(x^2) \;\Rightarrow\; y'=\dfrac{\sin(y^2)-2xy\cos(x^2)}{\sin(x^2)-2xy\cos(y^2)}$

21. $y\sin 2x=x\cos 2y \;\Rightarrow\; y\cdot\cos 2x\cdot 2+\sin 2x\cdot y'=x(-\sin 2y\cdot 2y')+\cos(2y)\cdot 1 \;\Rightarrow$

$\sin 2x\cdot y'+2x\sin 2y\cdot y'=-2y\cos 2x+\cos 2y \;\Rightarrow$

$y'(\sin 2x+2x\sin 2y)=-2y\cos 2x+\cos 2y \;\Rightarrow\; y'=\dfrac{-2y\cos 2x+\cos 2y}{\sin 2x+2x\sin 2y}$. When $x=\frac{\pi}{2}$ and $y=\frac{\pi}{4}$, we have

$y'=\dfrac{(-\pi/2)(-1)+0}{0+\pi\cdot 1}=\dfrac{\pi/2}{\pi}=\dfrac{1}{2}$, so an equation of the tangent line is $y-\frac{\pi}{4}=\frac{1}{2}(x-\frac{\pi}{2})$, or $y=\frac{1}{2}x$.

24. $x^2+2xy-y^2+x=2 \;\Rightarrow\; 2x+2(x\,y'+y\cdot 1)-2y\,y'+1=0 \;\Rightarrow\; 2x\,y'-2y\,y'=-2x-2y-1 \;\Rightarrow$

$y'(2x-2y)=-2x-2y-1 \;\Rightarrow\; y'=\dfrac{-2x-2y-1}{2x-2y}$. When $x=1$ and $y=2$, we have

$y'=\dfrac{-2-4-1}{2-4}=\dfrac{-7}{-2}=\dfrac{7}{2}$, so an equation of the tangent line is $y-2=\frac{7}{2}(x-1)$ or $y=\frac{7}{2}x-\frac{3}{2}$.

27. $2(x^2+y^2)^2=25(x^2-y^2) \;\Rightarrow\; 4(x^2+y^2)(2x+2y\,y')=25(2x-2y\,y') \;\Rightarrow$

$4(x+y\,y')(x^2+y^2)=25(x-y\,y') \;\Rightarrow\; 4y\,y'(x^2+y^2)+25yy'=25x-4x(x^2+y^2) \;\Rightarrow$

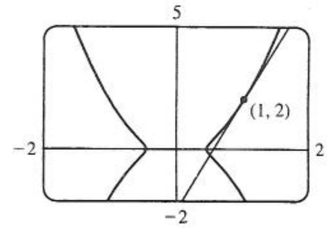
$y'=\dfrac{25x-4x(x^2+y^2)}{25y+4y(x^2+y^2)}$. When $x=3$ and $y=1$, we have $y'=\frac{75-120}{25+40}=-\frac{45}{65}=-\frac{9}{13}$,

so an equation of the tangent line is $y-1=-\frac{9}{13}(x-3)$ or $y=-\frac{9}{13}x+\frac{40}{13}$.

29. (a) $y^2 = 5x^4 - x^2 \quad\Rightarrow\quad 2y\,y' = 5(4x^3) - 2x \quad\Rightarrow\quad y' = \dfrac{10x^3 - x}{y}$.

So at the point $(1, 2)$ we have $y' = \dfrac{10(1)^3 - 1}{2} = \dfrac{9}{2}$, and an equation of the tangent line is $y - 2 = \frac{9}{2}(x - 1)$ or $y = \frac{9}{2}x - \frac{5}{2}$.

(b)



3.6

17. $y = (\tan^{-1} x)^2 \quad \Rightarrow \quad y' = 2(\tan^{-1} x)^1 \cdot \frac{d}{dx}(\tan^{-1} x) = 2\tan^{-1} x \cdot \frac{1}{1+x^2} = \frac{2\tan^{-1} x}{1+x^2}$

19. $y = \sin^{-1}(2x+1) \quad \Rightarrow$

$$y' = \frac{1}{\sqrt{1-(2x+1)^2}} \cdot \frac{d}{dx}(2x+1) = \frac{1}{\sqrt{1-(4x^2+4x+1)}} \cdot 2 = \frac{2}{\sqrt{-4x^2-4x}} = \frac{1}{\sqrt{-x^2-x}}$$

22. $f(x) = x\ln(\arctan x) \quad \Rightarrow \quad f'(x) = x \cdot \frac{1}{\arctan x} \cdot \frac{1}{1+x^2} + \ln(\arctan x) \cdot 1 = \frac{x}{(1+x^2)\arctan x} + \ln(\arctan x)$

27. $y = x\sin^{-1} x + \sqrt{1-x^2} \quad \Rightarrow$

$$y' = x \cdot \frac{1}{\sqrt{1-x^2}} + (\sin^{-1} x)(1) + \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x - \frac{x}{\sqrt{1-x^2}} = \sin^{-1} x$$

32. $\tan^{-1}(xy) = 1 + x^2 y \quad \Rightarrow \quad \frac{1}{1+x^2y^2}(xy' + y \cdot 1) = 0 + x^2y' + 2xy \quad \Rightarrow$

$$y'\left(\frac{x}{1+x^2y^2} - x^2\right) = 2xy - \frac{y}{1+x^2y^2} \quad \Rightarrow$$

$$y' = \frac{2xy - \dfrac{y}{1+x^2y^2}}{\dfrac{x}{1+x^2y^2} - x^2} = \frac{2xy(1+x^2y^2) - y}{x - x^2(1+x^2y^2)} = \frac{y(-1-2x-2x^3y^2)}{x(1-x-x^3y^2)}.$$

3.7

2. $f(x) = x \ln x - x \quad \Rightarrow \quad f'(x) = x \cdot \dfrac{1}{x} + (\ln x) \cdot 1 - 1 = 1 + \ln x - 1 = \ln x$

10. $f(t) = \dfrac{1 + \ln t}{1 - \ln t} \quad \Rightarrow \quad f'(t) = \dfrac{(1 - \ln t)(1/t) - (1 + \ln t)(-1/t)}{(1 - \ln t)^2} = \dfrac{(1/t)[(1 - \ln t) + (1 + \ln t)]}{(1 - \ln t)^2} = \dfrac{2}{t(1 - \ln t)^2}$

14. $F(y) = y \ln(1 + e^y) \quad \Rightarrow \quad F'(y) = y \cdot \dfrac{1}{1 + e^y} \cdot e^y + \ln(1 + e^y) \cdot 1 = \dfrac{ye^y}{1 + e^y} + \ln(1 + e^y)$

24. $f(x) = \ln \ln \ln x \quad \Rightarrow \quad f'(x) = \dfrac{1}{\ln \ln x} \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x}.$

$\text{Dom}(f) = \{x \mid \ln \ln x > 0\} = \{x \mid \ln x > 1\} = \{x \mid x > e\} = (e, \infty).$