MA 123 MIDTERM 1
THURSDAY, MAY 29 6PM

Instructions: Please write all of your work and solutions in the blue book provided. Only work appearing in the blue book will be graded. Please show all work and write clearly in order to receive full/partial credit. This is a closed-book test. Calculators and any other electronic devices are *not* permitted. This is a 50 minute exam.

Problem 1: Find the following limits. [20 pts]

(a.) $\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)$

(b.) $\lim_{h\to 0}\frac{\frac{1}{2\sqrt{a+h}}-\frac{1}{2\sqrt{a}}}{h}$

(c.) The following limit represents the derivative of some function f at $x=0$. State the function f.

$$\lim_{t\to -1}\frac{t^3+3t^2+3t+1}{t+1}$$

Problem 2: [20 pts] Find the horizontal and vertical asymptotes of the following function.

$$f(x)=\frac{2^x}{5-2^x}$$

Problem 3: [20 pts] Suppose the function f is defined as follows

$$f(x)=\begin{cases} cx^2+2x & \text{if } x<-2 \\ x^3-cx & \text{if } x\geq -2 \end{cases}$$

(a.) Find the value of the constant c such that the function $f(x)$ is continuous at $x=-2$.

(b.) Compute for any fixed c:

$$\lim_{h\to 0}\frac{c(x+h)^2+2(x+h)-(cx^2+2x)}{h}$$

(c.) Compute for any fixed c:

$$\lim_{h\to 0}\frac{(x+h)^3-c(x+h)-(x^3-cx)}{h}$$

(d.) We will say that the function $f(x)$ is *smooth* at $x = -2$ if the answers to parts (b.) and (c.) are identical when $x = -2$. Let c be the answer to part (a.). Is f *smooth* at $x = -2$?

Problem 4: [20 pts] Let

$$f(x) = \frac{2}{\sqrt{x}}$$

(a.) Find the equation of the tangent line at the point $(1, 2)$.

(b.) Find the equation of the line that goes through $(1, 2)$ and is perpendicular to the line found in (a.).

Problem 5: [20 pts] (a.) Let

$$f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Does $\lim_{x \to 0^+} f(x)$ exist? Justify your answer and compute the limit if necessary.

(b.) Now, suppose

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Does $\lim_{x \to 0^+} f(x)$ exist? Justify your answer and compute the limit if necessary.