MA 123 MIDTERM 2
THURSDAY, JUNE 12 6PM

Instructions: Please write all of your work and solutions in the blue book provided. Only work appearing in the blue book will be graded. Please show all work and write clearly in order to receive full/partial credit. This is a closed-book test. Calculators and any other electronic devices are *not* permitted. This is a 90 minute exam.

Problem 1: For each of the following, compute the derivative of y with respect to x.

(a.) $y = e^{\alpha x} \sin \beta x$, where α, β are some fixed constants.

(b.) $y = \frac{e^{\sqrt{\tan x}}}{\cos x}$

(c.) $\ln^2 x - 2xy + y^3 = c$, where c is a fixed constant.

Problem 2:

(a.) Find the equation of the tangent line to the curve $y = \sin^2 x + \sin x$ at the point $(0, 0)$.

(b.) Let $y = \sqrt{\frac{x^2+1}{x^2-1}}$. Use logarithmic differentiation to find the derivative of y with respect to x.

(c.) Evaluate the following limit and be sure to justify your answer.

$$\lim_{x \to 0^+} x \ln x$$

Problem 3:

A trough (V-shaped receptacle for drinking water) is 6 ft long and its ends have the shape of isosceles triangles that are 4 ft across at the top and have a height of 1 ft. Recall, an isosceles triangle has two sides with equal length. If the trough is being filled with water at a rate of 10 ft^3 per minute, how fast is the water level rising when the water is 0.5 ft deep? Note, the volume of the trough is $V = \frac{w}{2}hl$, where w is the width, h is the height and l is the length.

Problem 4:

Let $f(x) = \frac{x^2}{(x-2)^2}$. Sketch the function f. Along with your sketch, compute any vertical and/or horizontal asymptotes, local minimum and/or maximum values, intervals of concavity and points of inflection.