Turn in #1

The complex numbers are defined as

$$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}$$

where

$$\begin{aligned}(a + bi) + (c + di) &= (a + c) + (b + d)i \\ (a + bi)(c + di) &= (ac - bd) + (ad + bc)i\end{aligned}$$

and

$$\mathbb{R} \times \mathbb{R} = \{(a, b) \mid a, b \in \mathbb{R}\}$$

where

$$\begin{aligned}(a, b) + (c, d) &= (a + c, b + d) \\ (a, b)(c, d) &= (ac, bd)\end{aligned}$$

(a) We've said that $\mathbb{C}$ is a vector space over $\mathbb{R}$ and that the same is true for $\mathbb{R} \times \mathbb{R}$. The set $\{1, i\}$ is a basis for $\mathbb{C}$ over $\mathbb{R}$. Find a basis for $\mathbb{R} \times \mathbb{R}$ and determine if $\mathbb{C}$ and $\mathbb{R} \times \mathbb{R}$ are isomorphic as vector spaces? Explain.
(b) We know that $\mathbb{C}$ is a field. Is $\mathbb{R} \times \mathbb{R}$ a field? Is it a domain?
(c) Does $\mathbb{R} \times \mathbb{R}$ contain 'i'?
[10 points]