

BOSTON UNIVERSITY MATHEMATICS COLLOQUIUM

Configurations, arithmetic groups, cohomology, and stability

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Friday, March 28 at 4 pm
SCI 117

Abstract: Consider the following two objects:

- The congruence subgroup of level p in $\mathrm{SL}_n(\mathbf{Z})$; that is, the group of integral matrices congruent to 1 mod p ,
- The ordered configuration space of n points on a manifold M , which is to say, the space parametrizing ordered n -tuples of distinct points on M .

Each of these objects carries a natural action of the symmetric group S_n on n letters. (In the first case, this is by permuting the elements of the standard basis; in the second case, by permuting the points in the n -tuple.) What's more, each one is naturally described by cohomology groups H^i , which inherit the action and thus become representations of S_n . Although these examples are quite different, it turns out there is a general notion of stability which applies to both of these cases (and many other examples in representation theory, algebraic geometry, and combinatorics.) In some sense, each H^i is “the same” representation of S_n for all sufficiently large n . This implies, for instance, that the dimensions of these cohomology groups are (for sufficiently large n) polynomials in n . In the congruence subgroup context, our results refine a 2012 theorem of Putman. In the configuration space context, the result here refines a 2011 theorem of Church. The main ingredient is the theory of FI-modules, developed by myself, Tom Church, and Benson Farb, together with a Noetherianness theorem proved by the three of us and Rohit Nagpal: 1204.4533, 1210.4533.

From 2:30-3:00 pm in MCS 148, there will be a pre-colloquium talk by Jared Weinstein, “Representation theory of the symmetric group.” Abstract: We will review the classification of irreducible representations of the symmetric group on n letters using Young tableaux. After this we can introduce the notion of FI-module, which is the subject of the colloquium. In brief, an FI-module is a representation of “all the symmetric groups at once”. There will be a tea from 3:30-4:00 pm in MCS 144.