MR2514491 (2010k:11085) 11F75 (11F60 11F85)
Pollack, David (1-WESL-C); **Pollack, Robert [Pollack, Robert[2]]** (1-BOST-MS)
A construction of rigid analytic cohomology classes for congruence subgroups of $\mathrm{SL}_3(\mathbb{Z})$. (English summary)
Canad. J. Math. **61** (2009), *no. 3,* 674–690.

The article under review gives a constructive proof of a theorem of Ash and Stevens on the existence of rigid analytic cohomology classes for congruence subgroups Γ of $\mathrm{SL}_3(\mathbf{Z})$. These rigid analytic cohomology classes are cocycles on Γ with values in an appropriate space of p-adic distributions. Their interest lies in the fact that they are attached to p-adic families of modular forms on GL_3, and seem to encode much useful information about these families, such as their associated p-adic L-functions. A key idea in the author's construction is that the rigid analytic cohomology classes are characterised by their specialisation to a certain (fixed) weight k_0, together with the fact that they are eigenvectors for the Hecke operators at p (the so-called U_p-operator, which plays a key role in the theory). In the setting of ordinary families, such eigenclasses can be obtained by an iterative approach which involves applying the U_p operator repeatedly to a class which admits the desired specialisation in weight k_0. One of the key computational simplifications in the authors' algorithm arises from their adaptation of an idea of Matthew Greenberg to the setting of GL_3. Namely, in constructing the desired eigenclass, they observe that it is possible to work at the level of distribution-valued cochains rather than cocycles, because the U_p operator acts topologically nilpotently on the quotient of the former by the latter; thus the U_p-eigenvector that is obtained in the limit automatically satisfies a cocycle condition. This fact turns out to be very useful, because it is easier computationally to lift a weight k_0-cocycle to a distribution-valued cochain, for which no cocycle relation has to be enforced. It is worth insisting on the fact that, although rigid analytic cohomology classes may appear hard to compute a priori (for example, the associated distribution spaces are typically infinite-dimensional), the constructions explained in this article lead to an algorithm that is efficient in practice as well as in theory. In fact, the authors' approach, which builds on fundamental ideas of Stevens, also appears to be the best suited to working algorithmically with p-adic families of modular forms.

Reviewed by *Henri Darmon*

References

1. A. Ash, *Cohomology of congruence subgroups* $\mathrm{SL}(n,\mathbf{Z})$. Math. Ann. **249**(1980), no. 1, 55–73. MR0575448 (82f:22010)
2. A. Ash, D. Pollack, and G. Stevens, *Rigidity of p-adic cohomology classes of congruence subgroups of $GL(n,\mathbf{Z})$*. Proc. London. Math. Soc. **96**(2008), no. 2, 367–388. MR2396124 (2010a:11094)
3. A. Ash, D. Doud, and D. Pollack, *Galois representations with conjectural connections to arithmetic cohomology*. Duke Math. J. **112**(2002), no. 3, 521–579. MR1896473 (2003g:11055)

4. A. Ash, D. Grayson, and P. Green, *Computations of cuspidal cohomology of congruence subgroups of* $\mathrm{SL}(3, \mathbf{Z})$. J. Number Theory **19**(1984), no. 3, 412–436. MR0769792 (86g:11032)
5. A. Ash, G. Stevens, *p-adic deformations of cohomology classes of subgroups of* $\mathrm{GL}(n, \mathbb{Z})$. Journées Arithmétiques, (Barcelona, 1995) Collect. Math. **48**(1997), no. 1–2, 1–30. MR1464013 (98h:11064)
6. A. Ash and G. Stevens, *p-adic deformations of cohomology on* $\mathrm{GL}(n)$: *the non-ordinary case.* In preparation, draft available at http://math.bu.edu/people/ghs/research.d.
7. H. Darmon, *Integration on* $\mathcal{H}_p \times \mathcal{H}$ *and arithmetic applications.* Ann. of Math. **154**(2001), no. 3, 589–639. MR1884617 (2003j:11067)
8. H. Darmon and R. Pollack, *The efficient calculation of Stark-Heegner points via overconvergent modular symbols.* Israel J. Math. **153**(2006), 319–354. MR2254648 (2007k:11077)
9. M. Greenberg, *Lifting modular symbols of non-critical slope.* Israel J. Math. **161**(2007), 141–155. MR2350160 (2008j:11059)
10. R. Pollack and G. Stevens, *Overconvergent modular symbols and p-adic L-functions.* Preprint, available at http://math.bu.edu/people/rpollack/Papers.
11. R. Pollack and G. Stevens, *Critical slope p-adic L-functions.* In preparation, draft available at http://math.bu.edu/people/rpollack/Papers.
12. G. Stevens, *Rigid Analytic Modular Symbols.* Preprint, available at http://math.bu.edu/people/ghs/resea
13. M. Trifković, *Stark-Heegner points on elliptic curves defined over imaginary quadratic fields.* Duke Math. J. **135**(2006), no. 3, 415–453. MR2272972 (2008d:11064)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.