MAGIC 3-MANIFOLD, HORSESHOE BRAIDS AND MINIMAL DILATATION PROBLEM

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I an interested in surface dynamics, in particular the dynamics of *pseudo-Anosovs*. This note contains references of my talk at Boston University/Keio University workshop 2014. For an expository article of my talk, see [2]. Some slides (with a lot of pictures) related to my talk can be found at

`http://www.math.sci.osaka-u.ac.jp/~kin/publication/poster_Brown.pdf`
`http://www.math.sci.osaka-u.ac.jp/~kin/publication/poster_tokyo.pdf`

Motivations of my study come from the *minimal entropy (dilatation) problem* on pseudo-Anosovs. For this problem and related questions, see [2, Section 1]. The magic 3-manifold, denoted by N can be defined as a complement of a certain link with 3 components in the 3-sphere S^3. This manifold N plays an important role to study the minimal dilatation problem, see long introductions in [3, Section 1] and [4, Section 1]. The following is a main theorem in the talk.

Main theorem ([3]) We have algorithms to construct the followings. For each fibered class a of N,

(1) the monodromy $\Phi_a : F_a \to F_a$ of the fibration on N associated to a, and
(2) in the case a is primitive, a train track representative $\mathfrak{p}_a : \tau_a \to \tau_a$ of $\phi_a = [\Phi_a]$ whose incidence matrix is Perron-Frobenius.

For other references (e.g. theorems by Fried and by Thurston etc.) of my talk, see [3, References].

In the open problem session, I mentioned minimal dilatation problems on pseudo-Anosov *braids*. I explained why we need to care about *horseshoe braids* to study this problem. For the definition of horseshoe braids, see [5, Section 4.3] and [1, Section 4.1]. Questions which I posed in the session can be found in [2, Conjecture 4.2]. For other questions and conjectures on the minimal dilatation problem, see [2, Section 4].

References

[1] E. Hironaka and E. Kin, *A family of pseudo-Anosov braids with small dilatation*, Algebraic and Geometric Topology 6 (2006), 699-738.

[2] E. Kin, *Notes on pseudo-Anosovs with small dilatations coming from the magic 3-manifold*, Representation spaces, twisted topological invariants and geometric structures of 3-manifolds, RIMS Kokyuroku 1836 (2013), 45-64. available at `http://www.kurims.kyoto-u.ac.jp/~kyodo/kokyuroku/contents/pdf/1836-05.pdf`

[3] E. Kin, *Dynamics of the monodromies of the fibrations on the magic 3-manifold*, preprint, available at `http://www.math.sci.osaka-u.ac.jp/~kin/publication/monodromy.pdf`

[4] E. Kin, S. Kojima and M. Takasawa, *Minimal dilatations of pseudo-Anosovs generated by the magic 3-manifold and their asymptotic behavior*, Algebraic and Geometric Topology 13 (2013), 3537-3602.

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[5] E. Kin and M. Takasawa, *Pseudo-Anosov braids with small entropy and the magic* 3*-manifold*, Communications in Analysis and Geometry 19 Number 4 (2011), 1-54.

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