Forecasting Mortality Rates by Using Spatio-Temporal Data

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Abstract

The Lee-Carter Model is well known as the famous classical model to forecast mortality rates by Lee and Carter (1992).

We propose the extended model which is applicable Spatio-Temporal Data by using the Lee-Carter Model framework. Then, we forecast the mortality rates including regional effects.

Objectives

The purpose of this research is forecasting mortality rates every country or region. (This is one of the most important factor when we consider such as the food problem.)

The Mortality Rate Data (Japan)							
Age＼Year	1998	1999	2000	・・・・・	2008	2009	2010
0	0.381%	0.365%	0.350%	・・・・・	0.263%	0.261%	0.252%
1	0.068%	0.061%	0.055%	・・・・・	0.041%	0.040%	0.041%
2	0.034%	0.034%	0.033%	・・・・・	0.022%	0.020%	0.020%
⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
87	14.553%	14.581%	13.337%	・・・・・	12.413%	11.596%	12.424%
88	15.829%	16.624%	14.822%	・・・・・	12.839%	13.421%	13.818%
89	17.455%	17.712%	16.749%	・・・・・	15.749%	13.827%	15.564%

Fig.1) The Mortality Rate Data (Japan)

Log Mortality rates plot (2010)

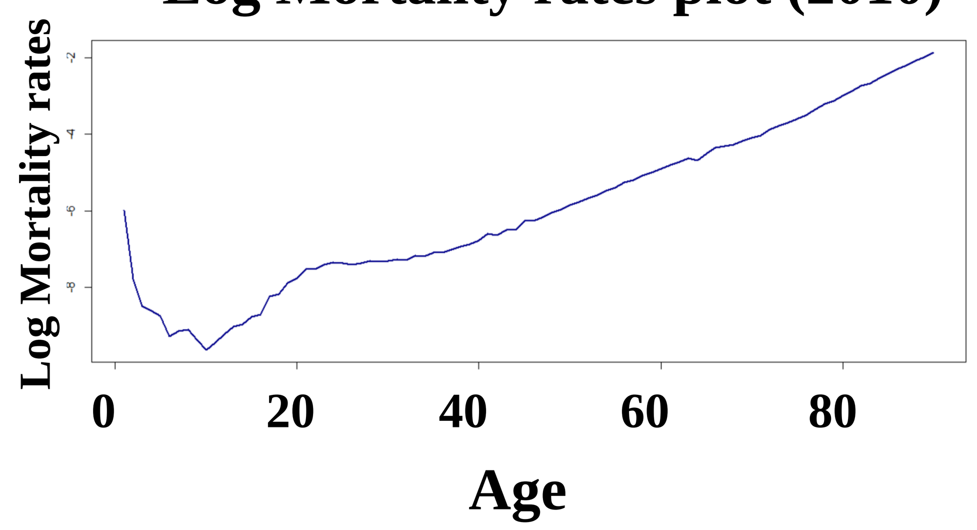

Fig.2) Log Mortality rates plot

Lee-Carter Model is well known as the famous classical model to forecast mortality rates by Lee and Carter(1992).
We explain Lee-Carter Model and consider Spatio-Temporal Model.

Lee-Carter Model

Lee-carter Model is given by this equation.

$$\boxed{\log m_{xt} = \alpha_x + \beta_x \kappa_t + \varepsilon_{xt}}$$

- m_{xt} : the mortality rate for age x in year t
- α_x : the age's effect
- κ_t : the year's effect
- β_x : the age's effect corresponding to κ_t
- ε_{xt} : observation error

Age
$$x = 0, \ldots, \omega$$
Year
$$t = 1, \ldots, T$$

$$\varepsilon_{xt} \sim N(0, \sigma^2)$$

$$\sum_{t=1}^{T} \kappa_t = 0 \quad \sum_{x=0}^{\omega} \beta_x = 1$$

We can forecast mortality rates $\hat{m}_{xT+h}$ by following steps.

Step1) Calculate the Least Squares Estimators $\hat{\alpha}_x, \hat{\beta}_x, \hat{\kappa}_t$ by using SVD(singular value decomposition)

Step2) Estimate $\hat{\kappa}_{T+h} (h = 1, \ldots)$ by fitting $\hat{\kappa}_1, \ldots, \hat{\kappa}_T$ to **ARIMA Model**

Step3) Forecast mortality rates $\hat{m}_{xT+h}$ by fitting $\hat{\alpha}_x, \hat{\beta}_x, \hat{\kappa}_{T+h}$ to Lee-Carter Model

※ARIMA $(p, 1, q)$ Model

The time series $\{Y_t\}$ is said to be ARIMA $(p, 1, q)$, if

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

where $\phi_1, \ldots, \phi_p$, $\theta_1, \ldots, \theta_q$ are fixed but unknown parameters.
$X_t = Y_t - Y_{t-1}$ and $\{\varepsilon_t\} \sim N(0, \sigma^2)$.

Analysis of the mortality rates data

We analyze the mortality rates data from 0 years old to 89 years old and from 1998 to 2010 in Japan (Fig.1). The results are Fig.3, Fig.4, Fig.5, and Fig.6. Fig.3 is $\hat{\alpha}_x$ plot.
It looks almost identical to log mortality rates plot in 2010 (Fig.2).

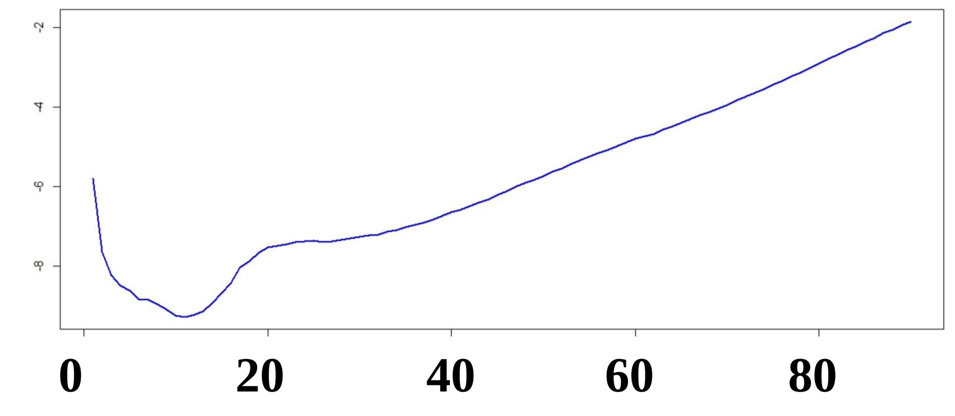

Fig.3) $\hat{\alpha}_x$ plot

Fig.4 is $\hat{\beta}_x$ plot and Fig.5 is $\hat{\kappa}_t$ plot. $\hat{\kappa}_t$ plot looks going down by little.

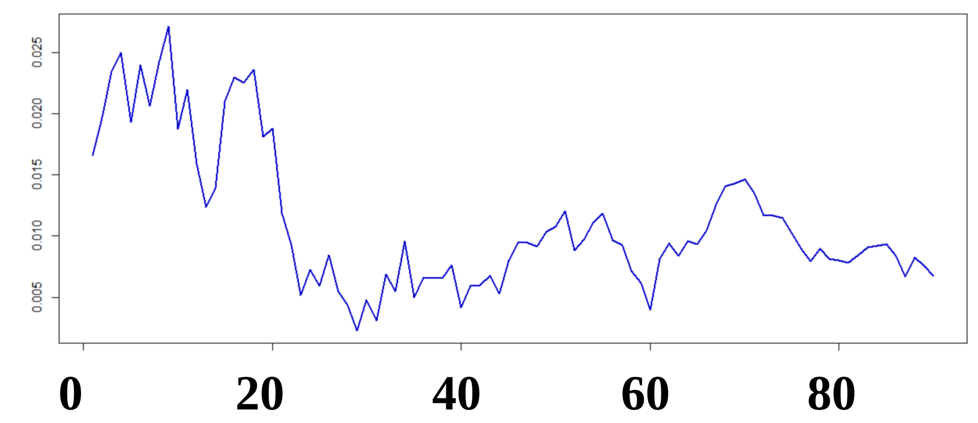

Fig.4) $\hat{\beta}_x$ plot

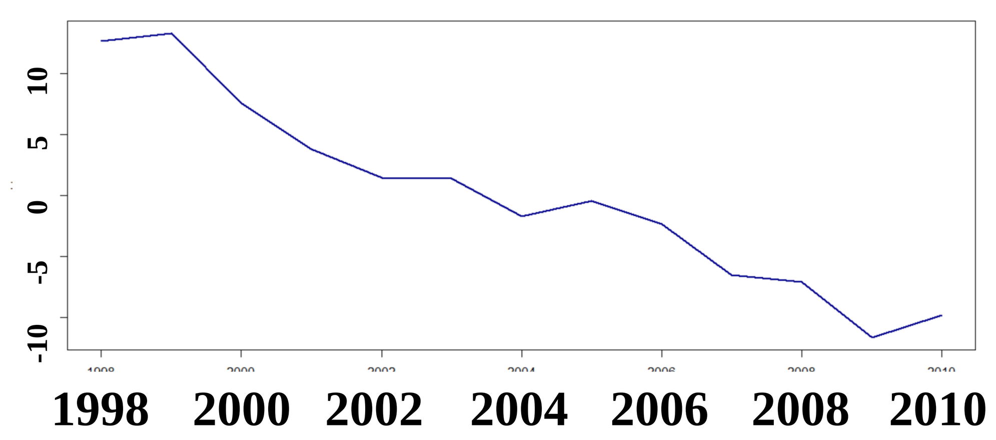

Fig.5) $\hat{\kappa}_t$ plot

Fig.6 is forecasting mortality rates plot from 2011 to 2060.

Japan: forecasting mortality rates (2011-2060)

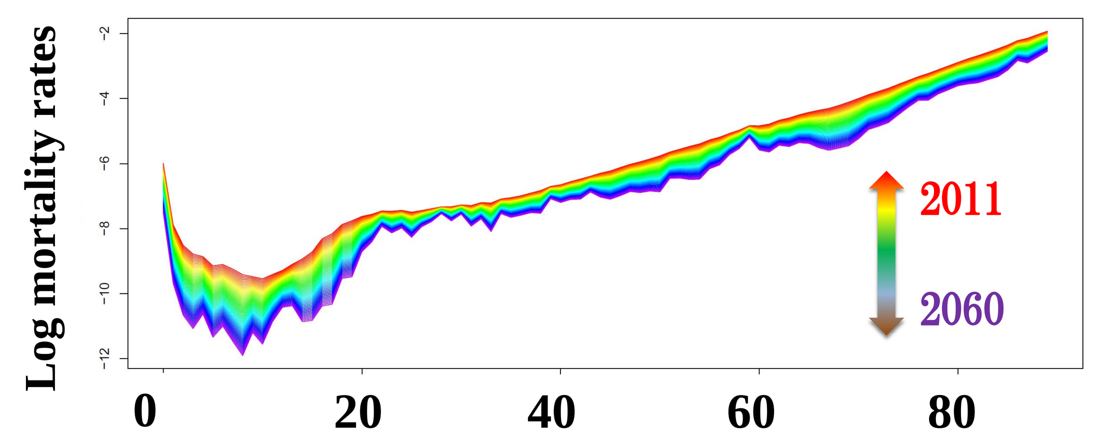

Fig.6) forecasting mortality rates plot

Spatio-Temporal Model

We propose the expanded model (Spatio-Temporal model) by using Lee-Carter model framework.
We calculate $\hat{\alpha}_x, \hat{\beta}_x, \hat{\kappa}_t$ in the same way as Step1 and estimate $\hat{\kappa}_{T+h}$ by fitting $\hat{\kappa}_1, \ldots, \hat{\kappa}_T$ to STARMA Model.

The time series and spatial process $\{\boldsymbol{Z}_t\}$ is said to be STARMA , if

$$\boldsymbol{Z}_t = \sum_{k=1}^{p} \sum_{j=1}^{\lambda_k} \phi_{kj} W_{kj} \boldsymbol{Z}_{t-k} - \sum_{l=0}^{q} \sum_{j=1}^{\mu_l} \theta_{lj} V_{lj} \boldsymbol{\varepsilon}_{t-l} + \boldsymbol{\varepsilon}_t$$

where $\boldsymbol{Z}_t = (Z(\boldsymbol{s}_1, t), \ldots, Z(\boldsymbol{s}_n, t))^{\mathrm{T}}$

We will consider the spatial weight matrices W_{kj}, V_{lj} as the future work.

Future Work

- Consider the spatial weight matrices W_{kj}, V_{lj}.
- Estimate the STARMA parameters ϕ_{kj}, θ_{lj} by maximum likelihood estimation method.
- Analyze the mortality data of each region by using Spatio-Temporal model.

References

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