Nonparametric Estimation for Optimal Dividend Barrier based on Empirical Process

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Introduction

Insurance company is exposed to uncertainties —when insured event occurs and how much the claim amount is. Lundberg (1903) modelled the fluctuation of the surplus of own company as

$$U(t) = u + ct - S(t),\ \ S(t) = \sum_{i=1}^{N(t)} X_i .$$

We suppose that the insurance company will refund an excess of the surplus ($U(t)$) over a previously determined barrier (b), as the dividends. We define the surplus process ($U_b(t)$) as follows:

$$U_b(t) = u + c\int_0^t \mathbb{I}\{U_b(s) < b\}ds - S(t)$$

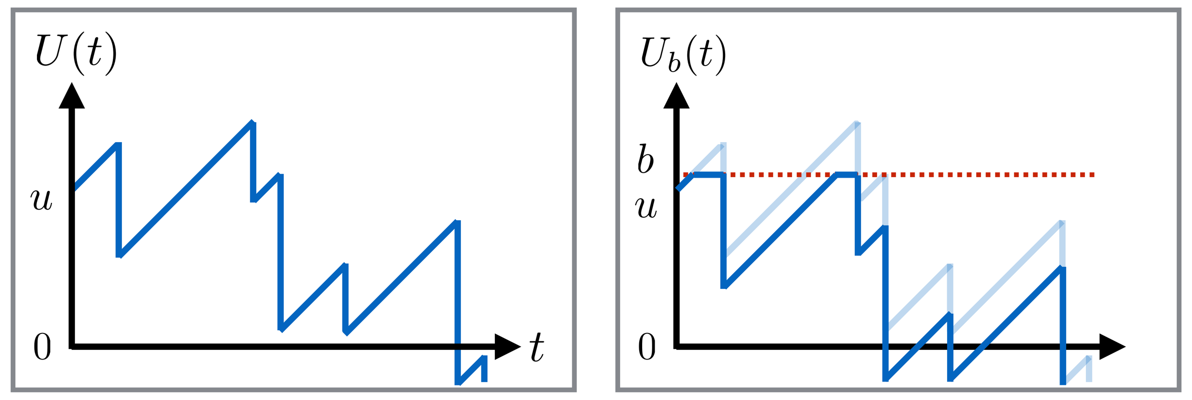


Fig.1 Sample paths of $U(t)$ and $U_b(t)$

What is the Optimal Dividend Barrier?

To define Optimal Dividend Barrier, we define $V(u,b)$ as the function aggregated expectation of present value of the dividends until ruin time:

$$V(u,b) = E\left[\int_0^{T_b} e^{-\delta t} dD_b(t)\right]$$

We define the Optimal Dividend Barrier by maximizing $V(u,b)$:

$$b^* := \underset{b\in\mathbb{R}^+}{\operatorname{argmax}} V(u,b)$$

Objectives

Our objectives in this research are as follows.

- To propose estimators
- To derive asymptotic properties (Consistency, Asymptotic normality, …)
- To perform simulation
- To analyze real data

In this poster, we focus on the consistency and simulation.
Firstly, we propose an estimator of $V(u,b)$, and define the estimator of b^* as its maximizer.
Secondly, we prove an asymptotic properties of estimators.
Lastly, we examine the convergence of estimators by simulation.

Estimator

Assume that we observe the data:

$$\{(X_i, \Delta_i T), i = 1, \dots, n\}$$

where X_i and $\Delta_i T$ are i th claim amount and inter-claim between i th claim and $i-1$ th claim, respectively.
We obtain sample paths by resampling from the data, and define the estimators of $V(u,b)$ and b^* as follows.

Definition

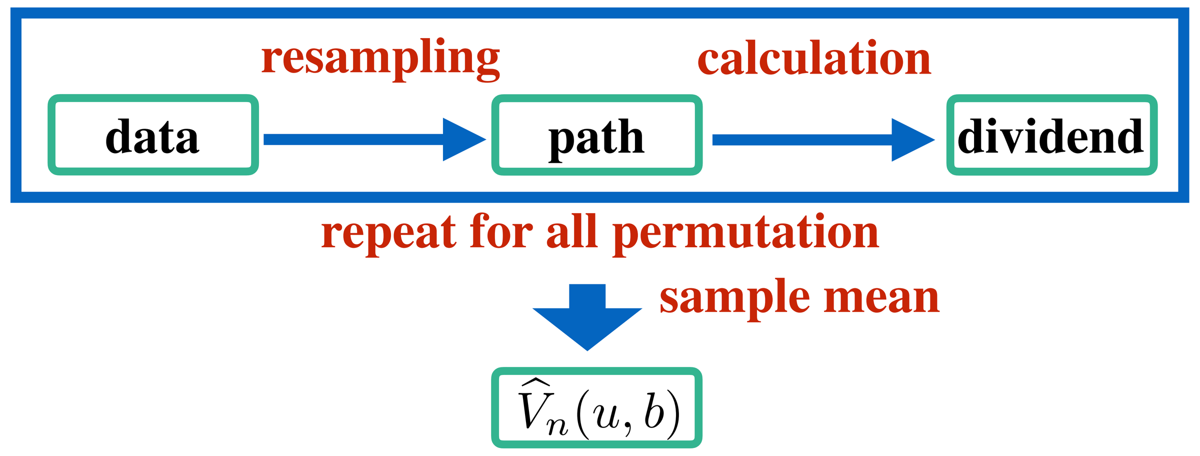
$$\widehat{V}_n(u,b) := E_n\left[\int_0^{T_b} e^{-\delta t} dD_{b,n}(t)\right],$$

$$\hat{b}_n^* := \underset{b\in\mathbb{R}^+}{\operatorname{argmax}} \widehat{V}_n(u,b)\,.$$

where $dD_{b,n}(t)$ denotes dividend amount for $(t, t+dt]$ under one of the sample paths:

$$\left\{(X_{i_j}, \Delta_{i_{j'}} T)\ ;\ i_j, i_{j'} \in \{1, \dots, n\}\right\}$$

E_n denotes sample mean for all permutations of $\{1, 2, \dots, n\}$.



Consistency

Under the regularity conditions, we prove uniform convergence of $\widehat{V}_n(u,b)$ and consistency of $\hat{b}_n^*$.
To prove consistency of $\hat{b}_n^*$, we use the following theorem.

Theorem (Consistency of M-estimator)

Assume the following three conditions,

- $^\forall\{b_n\} \in \mathbb{R}^+,\ \liminf_{n\to\infty} V(u,b_n) \geq V(u,b^*) \Rightarrow d(b_n, b^*) \to 0,$
- $\widehat{V}_n(u,\hat{b}_n^*) = \sup_{b\in\mathbb{R}^+} \widehat{V}_n(u,b) - o_P(1),$
- $\sup_{b\in\mathbb{R}^+} |\widehat{V}_n(u,b) - V(u,b)| \xrightarrow{P} 0.$

Then, $d(\hat{b}_n^*, b^*) \xrightarrow{P} 0.$

Proposition

$$\sup_{b\in\mathbb{R}^+} |\widehat{V}_n(u,b) - V(u,b)| \xrightarrow{P} 0$$

$$\hat{b}_n^* \xrightarrow{P} b^*$$

Simulation

Setting

observation : $\{(X_i, \Delta_i T), i = 1, \dots, n\}$

fix $N \leq n$, generating M paths

resampling : $\left\{(X_{i_j}, \Delta_{i_{j'}} T)\ ;\ i_j, i_{j'} \in \{1, \dots, N\}\right\}$

claim amount : $X_i \overset{i.i.d.}{\sim} Ex(1)$

inter-claim : $\Delta_i T \overset{i.i.d.}{\sim} Ex(1)$

premium rate : $c = 2$

interest rate : $\delta = 0.1$

Convergence of $\widehat{V}_n(u,b)$

Fig.2 shows that $\widehat{V}_n(u,b)$ converges to a unique smooth function of b as resampling size and number of sample paths go to infinity.

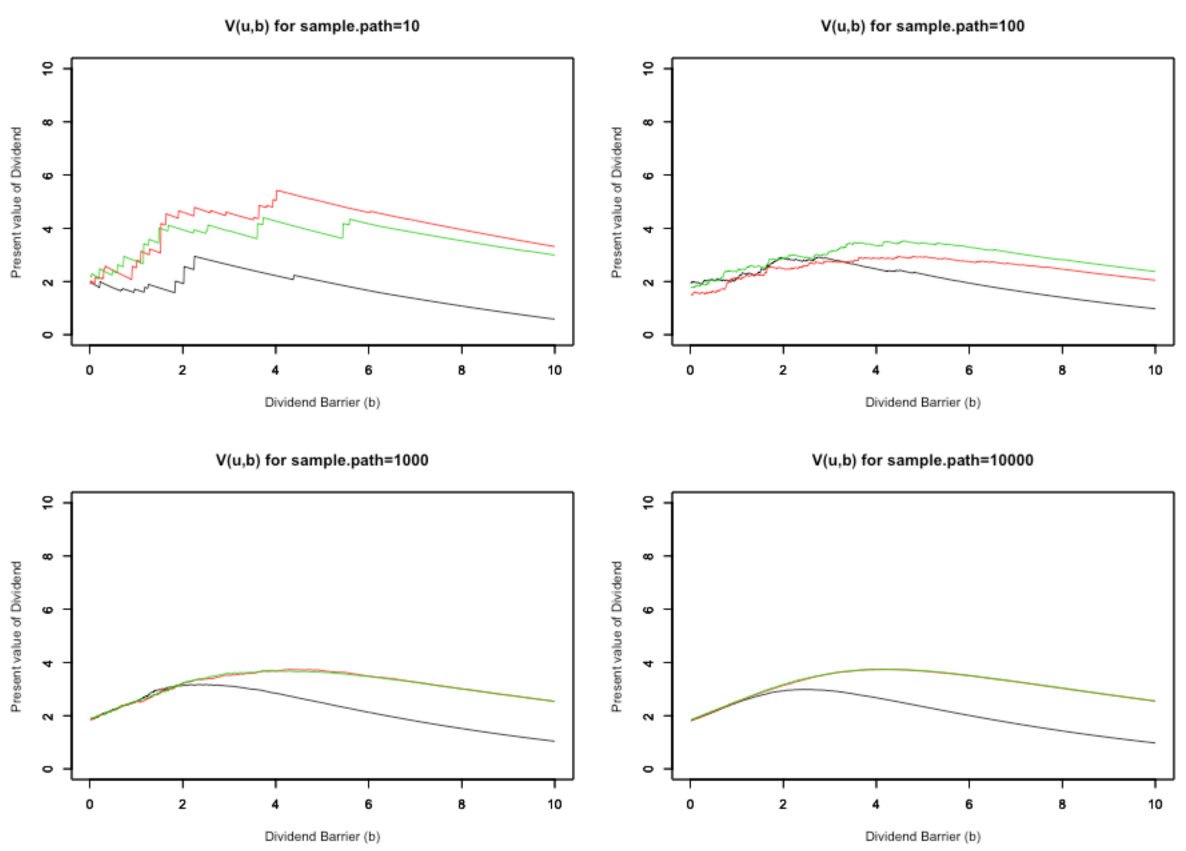


Fig.2 Convergence of $\widehat{V}_n(u,b)$

Fig.3 displays the behaviour of curves of various number of sample paths around the maximum of $\widehat{V}_n(u,b)$.

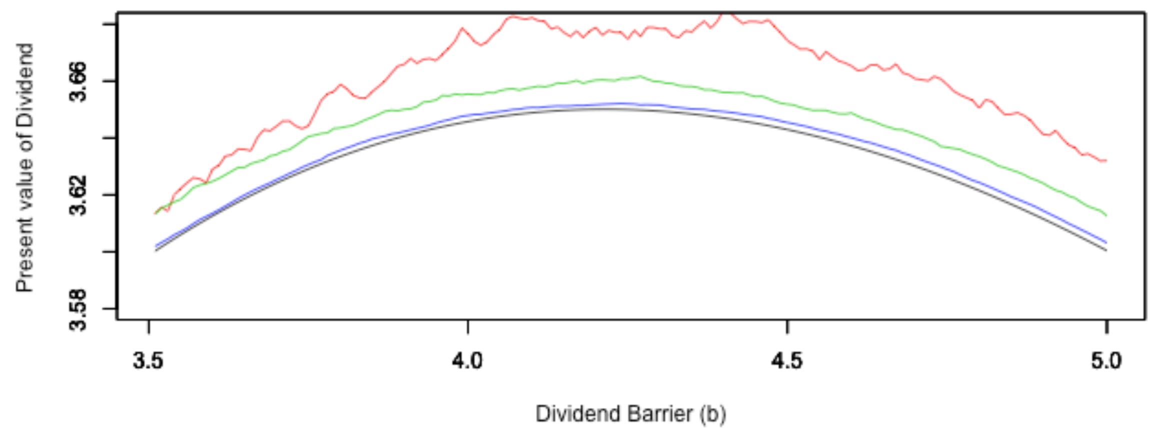


Fig.3 The behaviour of $\widehat{V}_n(u,b)$ around the maximizer

Convergence of $\hat{b}_n^*$

In Fig.4, we can observe the consistency of $\hat{b}_n^*$. Table.1 lists standard deviation of $\hat{b}_n^*$ for each sample size and number of sample paths.

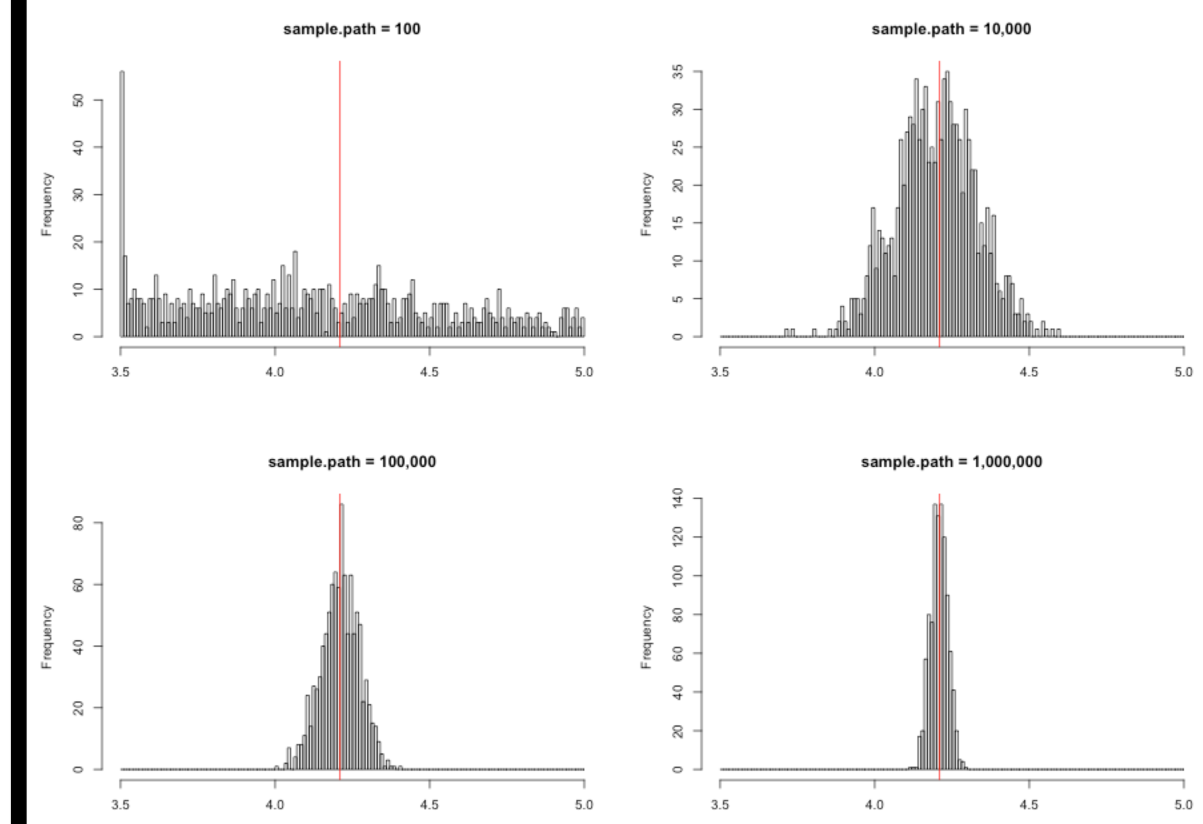


Fig.4 Convergence of $\hat{b}_n^*$

N \ M	100	10,000	100,000	1,000,000
100	0.4057	0.1361	0.0637	0.0280
1000	0.4134	0.1304	0.0619	0.0283

Table.1 Standard deviation of $\hat{b}_n^*$

Conclusion

- We proved consistency of $\hat{b}_n^*$.
- We showed the convergence of estimators by simulation.

Future work

- We derive asymptotic normality of $\hat{b}_n^*$.
- We try to perform simulation of the convergence of estimators with various cases.
- We analyze real data.

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