Squeezing Skyrmions

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Introduction to Skyrme Models

Motivation

- Low energy effective theory of hadrons - currently unknown
- Degrees of freedom of QCD:
  - high energy: quarks and gluons
  - low energy: hadrons
- One proposal: Skyrme model
  - primary fields are mesons
  - baryons (hadrons and nuclei) are realized as solitons
  - realizes unbroken symmetries
  - simplest case (two flavors): target space = SU(2) (isospin) matrix U
  - topological charge = baryon number

Generalised Skyrme Models

$$\mathcal{L}_{0246} = \boxed{\mathcal{L}_0} + \boxed{\mathcal{L}_2} + \boxed{\mathcal{L}_4} + \boxed{\mathcal{L}_6}$$

$$\boxed{\mathcal{L}_2} = -\lambda_2 \mathrm{Tr}\,(L_\mu L^\mu) \quad \boxed{\mathcal{L}_4} = \lambda_4 \mathrm{Tr}\,([L_\mu, L_\nu]^2) \quad \boxed{\mathcal{L}_6} = -(24\pi^2)^2 \lambda_6 \mathcal{B}_\mu \mathcal{B}^\mu$$

where

- left-invariant current $L_\mu = U^\dagger \partial_\mu U$
- $\boxed{\mathcal{L}_0}$ is a non-derivative part, i.e. a potential
- topological (baryon) currrent $\mathcal{B} = \frac{1}{24\pi^2}\epsilon_{ijk}\mathrm{Tr}\,(L_i L_j L_k)\ \mathrm{d}^3\mathrm{x}.$
- dimensionful coupling constants $\lambda_0, \lambda_2, \lambda_4, \lambda_6$

Note:

- Quadratic in first time derivatives $\Rightarrow$ *standard hamiltonian formulation*
- Poincare Symmetries
- $\boxed{\mathcal{L}_6}$ = Square of the topological (baryon) current!

$\mathcal{L}_{024}$ – Simplest Version

$$E_{024} = \boxed{\lambda_2} E_2 + \boxed{\lambda_4} E_4 + \boxed{\lambda_0} E_0 \,,$$

where the constants are

$$\boxed{\lambda_2} = \frac{1}{24\pi^2}, \quad \boxed{\lambda_4} = \frac{1}{12\pi^2}, \quad \boxed{\lambda_0} = \frac{1}{12\pi^2}$$

usual Skyrme potential

$$E_0 = \ \mathrm{Tr}\,(1 - U)$$

Finite energy configuration:

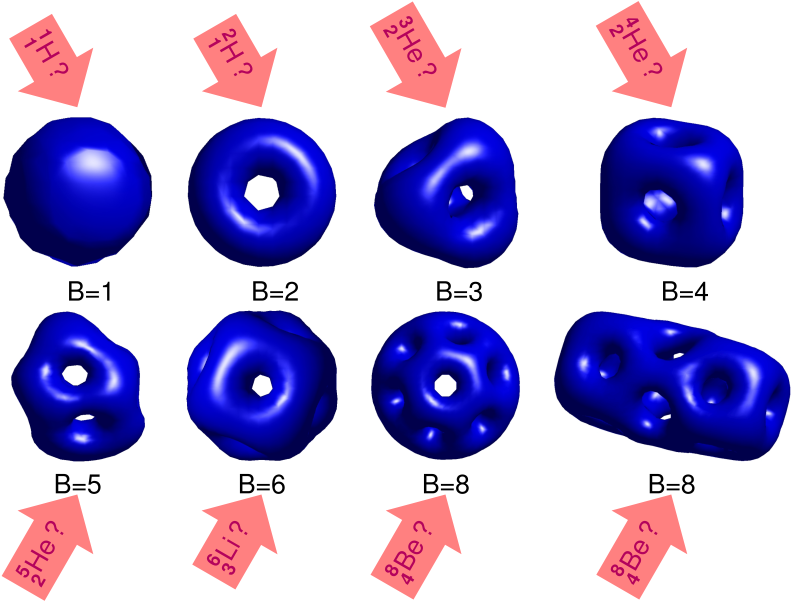
$$U(\mathbf{x}) \to \mathbb{1}_2 \text{ for } |\mathbf{x}| \to \infty$$

$\Rightarrow$ $U : S^3 \mapsto SU(2) \cong S^3$ $\Rightarrow$ $B \in \mathbb{Z} = \pi_3\,(SU(2))$

Note: Simplest models

- $\mathcal{L}_2 + \mathcal{L}_0$: excluded by *Derrick's scaling argument*
- $\mathcal{L}_4 + \mathcal{L}_0$: excluded by dynamics: no topological solitons

Classical Skyrmion Solutions



$\mathcal{L}_{024}$ – Hedgehogs

To make explicit the nonlinear pion theory, we write

$$U(x) = \boxed{\sigma(x)}\,\mathbb{1}_2 + i\,\boxed{\boldsymbol{\pi}(x)}\cdot\boldsymbol{\tau} \quad \text{and} \quad \boxed{\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} = 1}$$

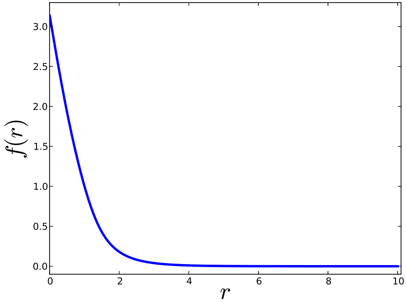
SU(2) constraint

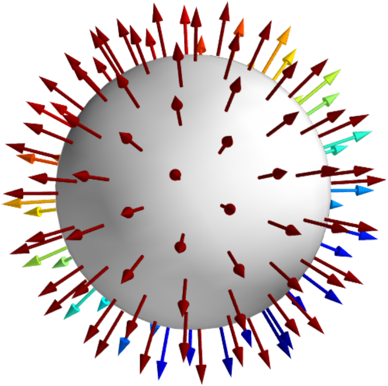
sigma field *pion field triplet*

- Hedgehog Field:

$$\sigma = \cos f(r)\,, \quad \boldsymbol{\pi} = \sin f(r)\hat{\boldsymbol{x}}\,,$$

where $f(0) = \pi$, $f(\infty) = 0$.





$\mathcal{L}_{024}$ – Sucesses of the Model

- energy spectra (Manko, Manton & Wood '07)
- spin and isospin states (Krusch '02)
- *E*2 Transitions (Haberichter, Lau & Manton)
- Nucleon-nucleon scattering (Foster & Krusch '15, Foster & Manton '15)
- States of Carbon-12 (Lau & Manton '14)
- charge-4 subunits (Battye, Manton & Sutcliffe '07)

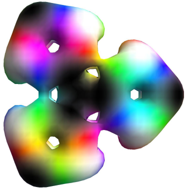


Figure : $B = 12$ triangle

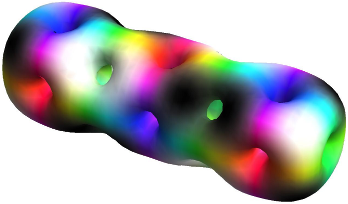


Figure : $B = 12$ linear chain

$\mathcal{L}_{024}$ – Challenges of the Model

- Unphysical large nuclear binding energies
- shell or crystal like densities
- Negative baryon densities (Foster & Krusch '13)
- Non BPS theory (Faddeev '76)
- Non-linear energy-baryon charge relation (Battye & Sutcliffe '97,'02,'05,'06)
- Rigid-Body quantization of Skyrmion solutions (Battye, Haberichter & Krusch '2014)

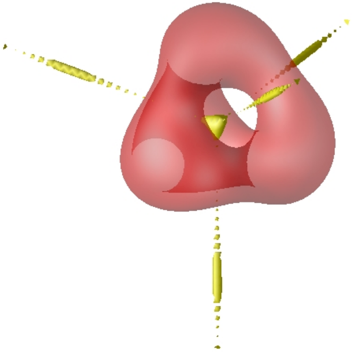


Figure taken from D. Foster, S. Krusch, J.Phys. A46 (2013) 265401.

$\mathcal{L}_{06}$ – BPS Skyrme Model

$$E_{06} = \boxed{\lambda_6} E_6 + \boxed{\lambda_0} E_0 \, ,$$

where common choices for the potential term are:

$$E_0 = \text{Tr}\,(1 - U) \quad \text{or} \quad \widetilde{E}_0 = E_0^2 \, .$$

Note:

- Derrick scaling $\Rightarrow$ $E_6 = E_0 \ldots$ compatible with BPS
- BPS (Bogomolny) bound
- ∞ many exact solutions saturating the BPS bound
- ∞ many symmetries
- Integrable: ∞ many conservation laws
- perfect fluid type description!

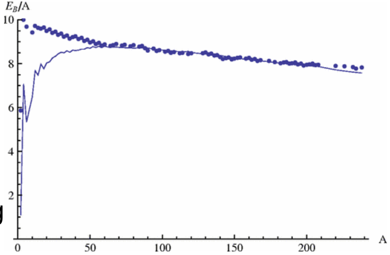


Figure taken from C. Adam, C. Naya, J. Sanchez-Guillen, and A. Wereszczynski Phys. Rev. Lett. 111, 232501.

$\mathcal{L}_{0246}$ – Near BPS Skyrme Model

$$E_{0246} = \epsilon \left(\boxed{\lambda_2} E_2 + \boxed{\lambda_4} E_4 + \boxed{\lambda_0} E_0 \right) + \boxed{\lambda_6} E_6 + \boxed{\tilde{\lambda}_0} \tilde{E}_0$$

where

- ϵ is a small parameter
- potential terms: $E_0 = \mathrm{Tr}\,(1 - U)$ and $\widetilde{E}_0 = E_0^2$.
- constants:

$$\boxed{\lambda_2} = \frac{1}{24\pi^2}, \quad \boxed{\lambda_4} = \frac{1}{12\pi^2} \quad \boxed{\lambda_0} = \frac{1}{12\pi^2},$$
$$\boxed{\lambda_6} = \lambda^2 \pi^4 \frac{1}{12\pi^2}, \quad \boxed{\tilde{\lambda}_0} = \frac{\mu^2}{12\pi^2}$$

- λ and μ^2 will be fixed later!

Note: The model is capable of producing skyrmions with very low binding energies. (Gillard, Harland & Speight '15)

Our Project

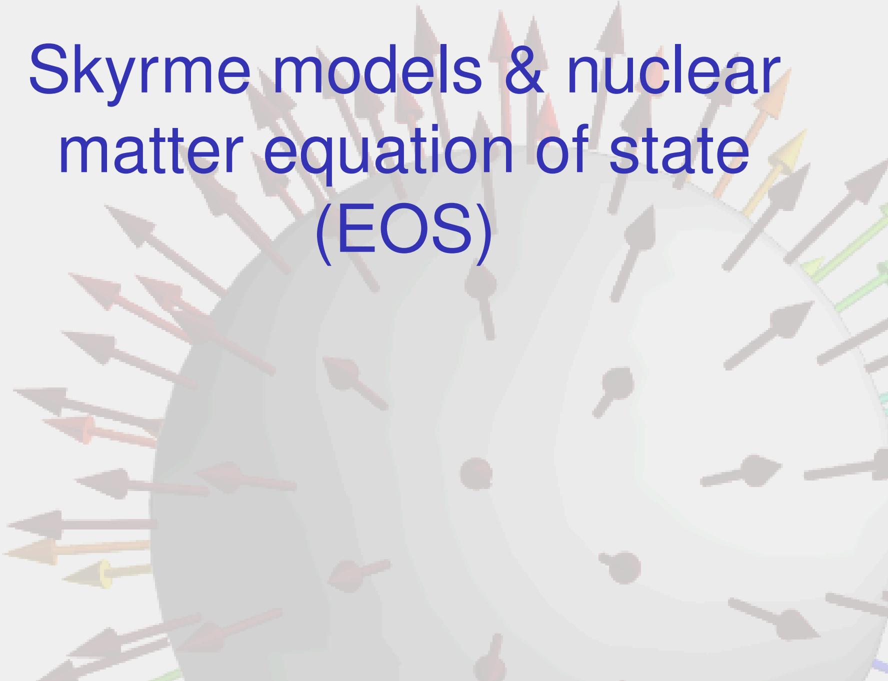
Aim

We want to investigate the *equation of state (EoS)* in the general Skyrme model with a special focus on near-BPS models.

Our Approach

- Find a defintion for average pressure and average chemical potential
- Derive an asymptotic EoS
- Learn about subleading terms in the EoS by using properties of generalised BPS Skyrme models
- Check numerically EoS for medium and large pressure assuming charge-1 hedgehog configurations
- Compare EoS with with the Walecka model

Skyrme models & nuclear matter equation of state (EOS)



Average Pressure & Average Chemical Potential – I

- Thermodynamical Relations have to be fulfilled:

$$\left(\frac{\partial E}{\partial V}\right)_B = -P, \qquad \left(\frac{\partial E}{\partial B}\right)_V = \bar{\mu}$$

- We define the generalized *step-function potential*:

$$\tilde{\Theta}(U) = \begin{cases} 1 & \text{for} \quad U \neq 1 \\ 0 & \text{for} \quad U = 1 \end{cases}$$

- We define the locus set of the skyrmion $U_0(\vec{x})$:

$$\Omega = \{\vec{x} \in \mathbb{R}^3 \mid U_0^*(\tilde{\Theta}(U)) \equiv \tilde{\Theta}(U_0(\vec{x})) = 1\}$$

- Skyrme static energy functional

$$E(V, P, B, \bar{\mu}) = \int d^3x\, \varepsilon[U] + \boxed{P} \left(\int d^3x\, \tilde{\Theta}(U(\vec{x})) - V\right) - \boxed{\bar{\mu}} \left(\int d^3x \mathcal{B}_0 - B\right)$$

- $\boxed{P}$: all possible solution must have volume V, i.e. $\int d^3x \tilde{\Theta}(U(\vec{x})) = \int_\Omega d^3x = V$.
- $\boxed{\bar{\mu}}$: all possible solution must have charge B

Average Pressure & Average Chemical Potential – II

Check: $\boxed{P}$ fulfills *average pressure definition*:

- Scaling transformations:

$$x^i \to e^\lambda x^i = (1+\lambda)x^i\,, \quad \delta_\lambda U = \lambda x^i \partial_i U\,.$$

- Energy functional under scaling transformation:

$$\delta_\lambda E = -\lambda \int T_{ii} d^3x + 3\lambda \boxed{P} \int d^3x\, \tilde{\Theta}(U(\vec{x}))\,.$$

- Any solution of the Euler-Lagrange equations is a stationary point $\Rightarrow \delta_\lambda E = 0$:

$$\boxed{P = \frac{\frac{1}{3}\int_\Omega T_{ii} d^3x}{\int_\Omega d^3x}} \quad \checkmark$$

Remarks:

- Compact domain Ω *is not* uniquely determined by $\boxed{P}$.
- *Thermodynamical volume* $=$ *geometrical volume* of a topological soliton. $\Rightarrow \bar{\varepsilon} = E/V = 0$ in the zero pressure limit!

Topological bounds

- Manton's *strain tensor formulation*:

$$D_{jk} = -\frac{1}{2}\mathrm{tr}\,(R_j R_k) \ .$$

- $D_{jk} =$ symmetric, positive 3×3 matrix with EVs $\tilde{\lambda}_1^2, \tilde{\lambda}_2^2, \tilde{\lambda}_3^2$.
- Use rescaled EVs $\lambda_i = \tilde{\lambda}_i / \sqrt[3]{2\pi^2}$.

$$\begin{aligned} E_4 &= 3\int_{\mathcal{M}} \Omega_{\mathcal{M}} \frac{1}{3}\left(\lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_1^2\lambda_3^2\right) \geq 3\int_{\mathcal{M}} \Omega_{\mathcal{M}} \left(\lambda_1^4\lambda_2^4\lambda_3^4\right)^{\frac{1}{3}} \\ &= 3\int_{\mathcal{M}} \Omega_{\mathcal{M}} |\mathcal{B}_0|^{\frac{4}{3}} \geq \frac{3}{\mathrm{Vol}_{\mathcal{M}}^{\frac{1}{3}}} \left(\left|\int_{\mathcal{M}} \Omega_{\mathcal{M}} \mathcal{B}_0\right|\right)^{\frac{4}{3}} = \frac{3}{\mathrm{Vol}_{\mathcal{M}}^{\frac{1}{3}}} |B|^{\frac{4}{3}} \ . \end{aligned}$$

$$\begin{aligned} E_6 &= \int_{\mathcal{M}} \Omega_{\mathcal{M}} \lambda_1^2\lambda_2^2\lambda_3^2 = \int_{\mathcal{M}} \Omega_{\mathcal{M}} \left(\mathcal{B}_0\right)^2 \\ &\geq \frac{1}{\mathrm{Vol}_{\mathcal{M}}} \left(\int_{\mathcal{M}} \Omega_{\mathcal{M}} \mathcal{B}_0\right)^2 = \frac{1}{\mathrm{Vol}_{\mathcal{M}}} |B|^2 \ . \end{aligned}$$

Asymptotic EoS – I

Topological bounds for the quartic and sextic terms:

$$E_4 = \frac{1}{16}\int d^3x \; \mathrm{Tr}\,[L_i, L_j]^2 \geq 3(2\pi^2)^{4/3}\frac{B^{4/3}}{V^{1/3}}$$

$$E_6 = \int d^3x \; (\epsilon^{ijk}\,\mathrm{Tr} L_i L_j L_k)^2 \geq \frac{B^2}{V}\,.$$

Asymptotic static energy for any Skyrme model can be approximated:

$$E = \lambda_2 E_2 + \lambda_4 E_4 + \lambda^2\pi^4 E_6 + \lambda_0 E_0 \geq \lambda_4 3(2\pi^2)^{4/3}\frac{B^{4/3}}{V^{1/3}} + \pi^4\lambda^2\frac{B^2}{V} \geq \pi^4\lambda^2\frac{B^2}{V}$$

Asymptotic formula for the energy:

$$\boxed{E = \pi^4\lambda^2\frac{B^2}{V} + \alpha\frac{B^{4/3}}{V^{1/3}} + o(V^{-1/3}) \quad \text{for} \quad V \to 0}$$

with

$$\boxed{\alpha \geq 3(2\pi^2)^{4/3}\lambda_4}$$

Asymptotic EoS – II

- Average pressure

$$P = \pi^4 \lambda^2 \bar{\rho}_B^2 + \frac{\alpha}{3} \bar{\rho}_B^{4/3} + o(\bar{\rho}_B^{4/3})$$

- Average baryon chemical potential

$$\bar{\boldsymbol{\mu}} = 2\pi^4 \lambda^2 \bar{\rho}_B + \frac{4\alpha}{3} \bar{\rho}_B^{1/3} + o(\bar{\rho}_B^{1/3})$$

  where $\bar{\rho}_B = B/V$ is the average baryon (particle) density.

- Average energy density

$$\bar{\varepsilon} = \pi^4 \lambda^2 \bar{\rho}_B^2 + \alpha \bar{\rho}_B^{4/3} + o(\bar{\rho}_B^{4/3})$$

- **Note:** BPS Skyrme model asymptotic behaviour in the leading approximation:

$$P = \pi^4 \lambda^2 \bar{\rho}_B^2 \,, \quad \bar{\boldsymbol{\mu}} = 2\pi^4 \lambda^2 \bar{\rho}_B \,, \quad \bar{\varepsilon} = \pi^4 \lambda^2 \bar{\rho}_B^2$$

  EOS:

$$\bar{\varepsilon} = P$$

Asymptotic EoS – The Main Findings

1. The sextic term gives the main contribution in the high pressure limit. This means that this term should not be omitted as far as dense nuclear matter is considered. The asymptotic equation of state always has a universal (potential independent) form $\bar{\varepsilon} = P$.
2. The quartic, usual Skyrme term, gives a subleading contribution which modifies the equation of state at moderate pressures. The functional dependence is known, which is not the case for the multiplicative constant α, for which we have derived a lower bound.
3. The potential and the sigma model part give contributions which are even subleading in comparison to the E_4 contribution. On the other hand, they may be significant close to nuclear saturation density.

The BPS Fluid Toy Models – I

Aim

We want to learn how a specific number of derivatives can change the equation of state

Skyrme (quartic) term

$$\mathcal{L}_4 = \lambda_4 \mathrm{Tr}\left([L_\mu, L_\nu]^2\right) \longrightarrow \mathcal{L}_4^f = \lambda_4 (\mathcal{B}^\mu \mathcal{B}_\mu)^{2/3}$$

which now is represented by a "*four derivative*" term (in the sense that under Derrick scaling $x^\mu \to \Lambda x^\mu$ it scales like Λ^{-4})

- energy density & pressure:

$$\varepsilon = \lambda_4 \rho_B^{4/3}, \quad P = \rho_B \frac{\partial \varepsilon}{\partial \rho_B} - \varepsilon = \frac{1}{3}\lambda_4 \rho_B^{4/3},$$

- EoS:

$$\boxed{P = \frac{1}{3}\varepsilon}$$

The BPS Fluid Toy Models – II

Sigma model term

$$\mathcal{L}_2 = -\lambda_2 \mathrm{Tr}\,(L_\mu L^\mu) \quad \longrightarrow \quad \mathcal{L}_2^f = -\lambda_2 (\mathcal{B}_\mu \mathcal{B}^\mu)^{1/3}$$

- energy density & pressure:

$$\varepsilon = \lambda_2 \rho_B^{2/3}\,, \quad P = -\frac{\lambda_2}{3}\rho_B^{2/3}$$

- EoS: $\boxed{P = -\frac{1}{3}\varepsilon.}$

Potential term

- energy density & pressure: $\varepsilon = \mathcal{U}, \quad P = -\mathcal{U}$
- EoS: $\boxed{P = -\varepsilon}$

The BPS Fluid Toy Models – III

Energy density-baryon density relation which can be valid not only for asymptotically high pressure but also in a medium pressure regime

$$\bar{\varepsilon} = \pi^4 \lambda^2 \bar{\rho}_B^2 + \alpha \bar{\rho}_B^{4/3} + \beta + \tilde{\beta}\ \bar{\rho}_B^{2/3} + O(1)$$

This leads, for the charge one sector, to the following energy-volume formula

$$\boxed{E = \pi^4 \lambda^2 \frac{1}{V} + \alpha \frac{1}{V^{1/3}} + \beta V + \tilde{\beta}\ V^{1/3} + O(1)}$$

Numerical Results for medium and large pressure regime

Calibration

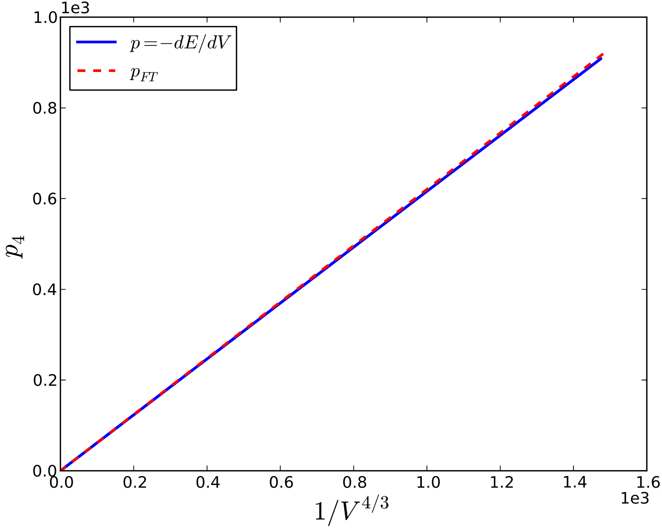
Adkins and Nappi (AN)

Adjust the parameters e, F_π, m_π to fit the hadron masses: N, Δ, π

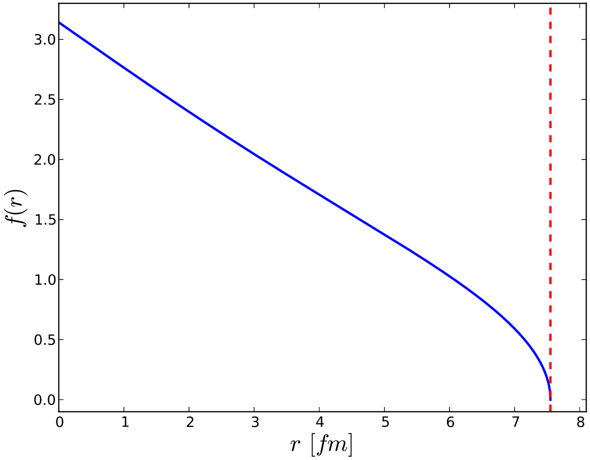
$$\left[\frac{F_\pi}{4e_{\mathrm{Sky}}}\right] = 12\pi^2 \times 5.58\,\mathrm{MeV}\,, \quad \left[\frac{2}{e_{\mathrm{Sky}} F_\pi}\right] = 0.755\,\mathrm{fm}$$

$$\hbar = 46.8\,, \quad e_{\mathrm{Sky}} = 4.84, \quad F_\pi = 108\,\mathrm{MeV}\,, \quad m_\pi = 138\,\mathrm{MeV}\,.$$

Check: field-theoretical P = thermodynamical P



E_4 – Profile function



E_4

- Energy integral

$$\mathrm{E}_4 = \lambda_4 E_4 = \frac{1}{12\pi^2} 4\pi \int_0^R drr^2 \left(2\frac{\sin^2 f}{r^2} f'^2 + \frac{\sin^4 \xi}{r^4} \right).$$

- Scale transformation $r \to \Lambda r$:

$$\mathrm{E}_4[V] = \frac{1}{V^{1/3}} \mathrm{E}_4[V=1] \equiv \alpha \frac{1}{V^{1/3}} \quad \Rightarrow p = \frac{\alpha}{3} \frac{1}{V^{4/3}}$$

- Numerics:

$$\alpha = 1.853 \quad \text{or} \quad \alpha = 924.4 \text{ MeV fm}$$

- Analytic:

$$\alpha = 3(2\pi^2)^{4/3}/(12\pi^2) = 1.351$$

- Density-pressure equation of state:

$$\bar{\varepsilon} = 3p.$$

➡ The bound *is not* saturated!

E_4

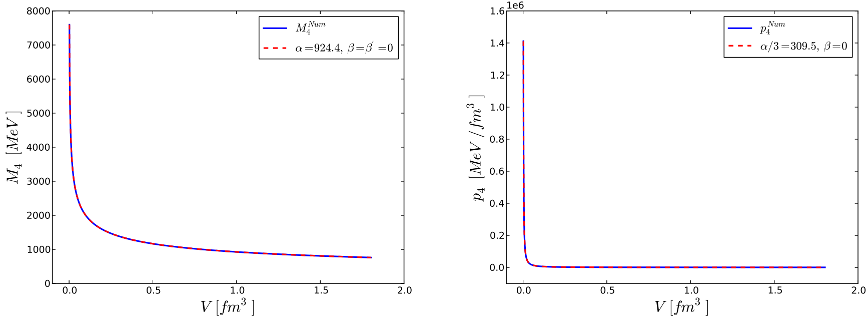
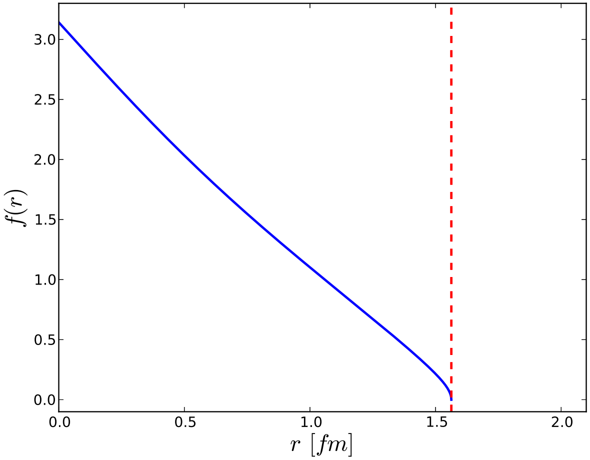


Figure : Energy and pressure of the B=1 skyrmion as a function of the volume for E_4 model

E_{04} – Profile function



E_{04}

- Size of compact charge one skyrmion at equilibrium:

$$R = 2.07 \quad \text{or} \quad R = 1.563 \text{ fm},$$

  This sets the maximum volume to $V_{max} = 15.9 \text{ fm}^3$.

- Asymptotic energy formula

$$\text{E} = \alpha \frac{1}{V^{1/3}} + \tilde{\beta} V^{1/3} + \beta V + o(V)$$

  with parameters

$$\alpha_{04} = 924.6 \text{ MeV fm}, \;\; \beta_{04} = 13.4 \text{ MeV fm}^{-3}, \;\; \tilde{\beta}_{04} = 0.$$

⇒ Mass of the equilibrium skyrmion: $M_{04} = 580$ MeV

⇒ 3.5% above the true mass

E_{04}

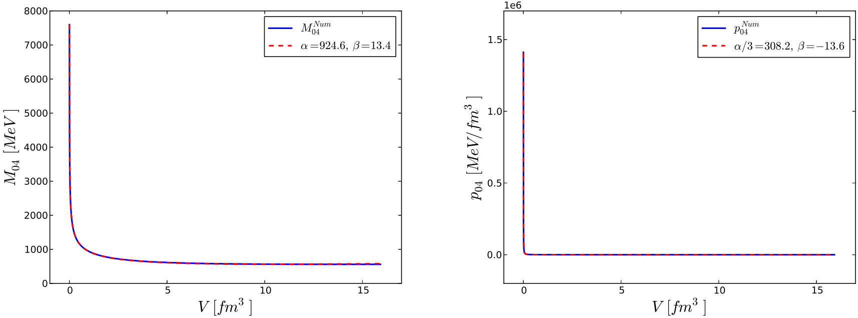
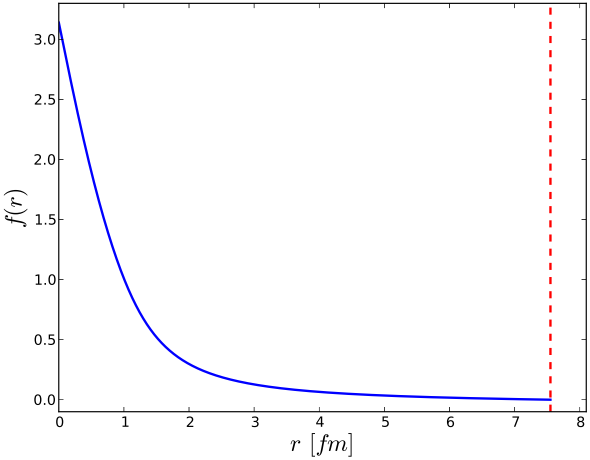


Figure : Energy and pressure of the B=1 skyrmion as a function of the volume for E_{04} model

E_{24} – Profile function



E_{24}

- The equilibrium solution in the charge one sector is an infinitely extended skyrmion.
- We find:

$$\alpha_{24} = 924.1 \text{ MeV fm}, \quad \beta_{24} = 0, \quad \tilde{\beta}_{24} = 311.5 \text{ MeV fm}^{-1}$$

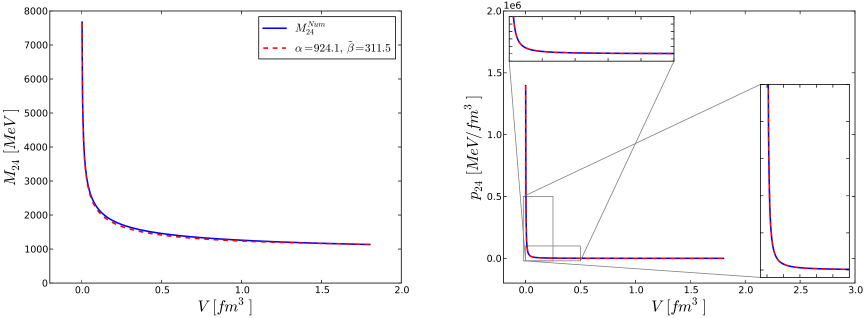
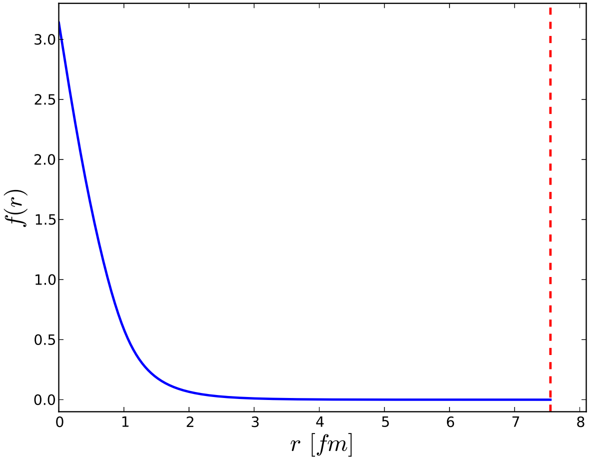
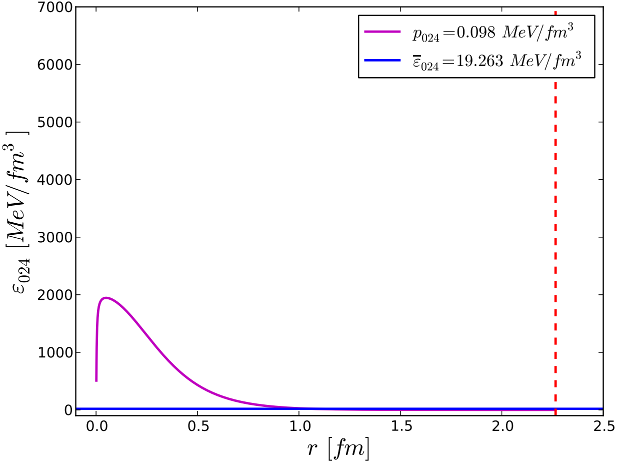


Figure : Energy and pressure of the B=1 skyrmion as a function of the volume for E_{24} model

E_{024} – Profile function



E_{024} – Energy Density Profiles



E_{024}

- Fitting parameters:

$$\alpha_{024} = 924.1 \text{ MeV fm}, \quad \beta_{024} = -106.1 \text{ MeV fm}^{-3},$$
$$\tilde{\beta}_{024} = 458.0 \text{ MeV fm}^{-1}$$

- Subtracting leading term from numerical energy gives *effective mass-volume formula*

$$E_{024} = \alpha \frac{1}{V^{1/3}} + \gamma V^{1/5} + o(V^{1/5}),$$

where $\gamma = 346.1 \text{MeV fm}^{-1/5}$.

⇒ strong mixing between the sigma model, potential and quartic term

⇒ **Note**: $\alpha = 1.853$ in Skyrme units (or $\alpha = 924.4$ MeV)
Kutschera et. al.: $\alpha_{kut} = 1.837$

E_{024}

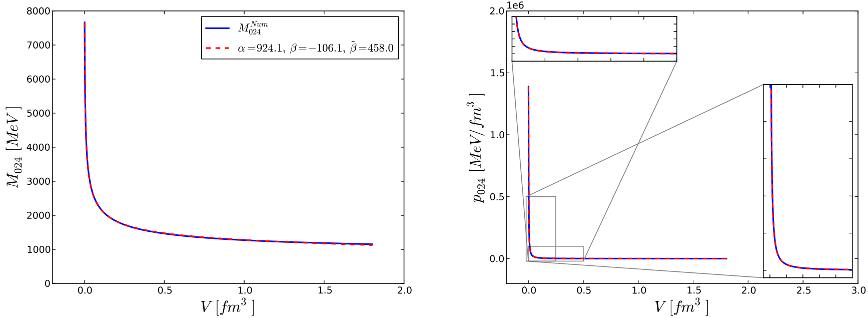


Figure : Energy and pressure of the B=1 skyrmion as a function of the volume for E_{024} model

E_{024}

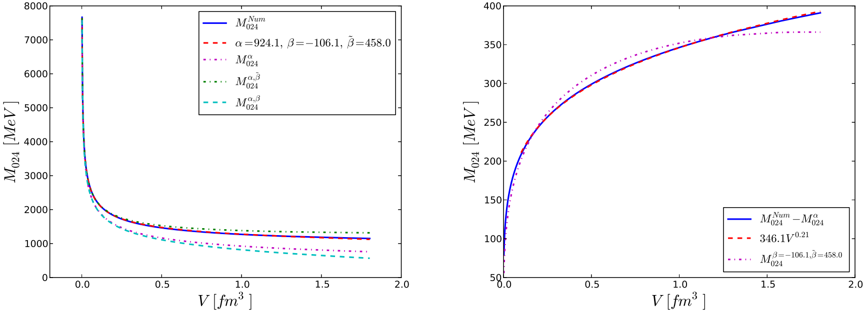


Figure : Energy of the B=1 skyrmion as a function of the volume for E_{024} model

E_{024} – EOS

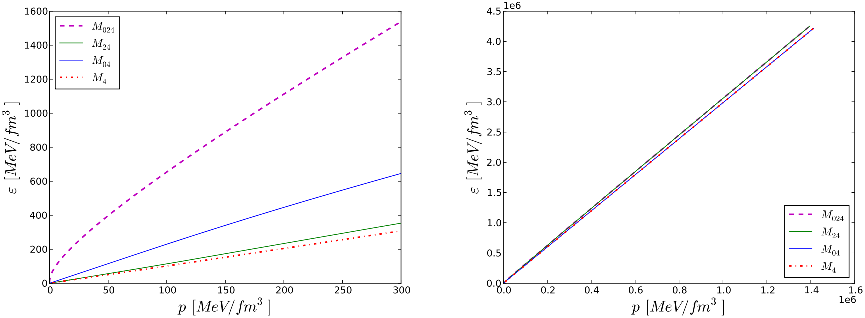


Figure : Equation of state for E_{024} model

The BPS model – E_{06} Model – I

- Analytical Solution:

$$\frac{\lambda}{2r^2}\sin^2 f f' = -\mu\sqrt{\tilde{E}_0 + P\frac{12\pi^2}{\mu^2}}\,.$$

- total energy E and volume V at a given pressure P:

$$E(P) = \frac{2\pi}{12\pi^2}\lambda\mu\int_0^\pi d\xi\,\sin^2 f\,\frac{8(1-\cos f)^2 + P\frac{12\pi^2}{\mu^2}}{\sqrt{4(1-\cos f)^2 + P\frac{12\pi^2}{\mu^2}}}$$

$$V(P) = 2\pi\frac{\lambda}{\mu}\int_0^\pi df\,\sin^2 f\,\frac{1}{\sqrt{4(1-\cos f)^2 + P\frac{12\pi^2}{\mu^2}}}$$

- mean-field equation of state

$$\bar{\varepsilon} = P + \tilde{\mu}^2\left(\frac{5}{2}\frac{{}_3F_2[\{\frac{1}{2},\frac{7}{4},\frac{9}{4}\},\{\frac{5}{2},3\},-\frac{4\tilde{\mu}^2}{P}]}{{}_3F_2[\{\frac{1}{2},\frac{3}{4},\frac{5}{4}\},\{\frac{3}{2},2\},-\frac{4\tilde{\mu}^2}{P}]}\right),$$

where ${}_pF_q[\{a_1,..,a_p\},\{b_1,..,b_q\},z]$ is a generalised hypergeometric function, and $\tilde{\mu}^2 = 1/3\pi^2$.

The BPS model – E_{06} Model – II

- Mean-field EOS in the asymptotic region

$$E = \tilde{\alpha}\frac{1}{V} + \beta_\infty V + o(V),$$

where

$$\tilde{\alpha} = \frac{1}{12\pi^2}\lambda^2\pi^4 = \frac{\pi^2}{3} = 935.6 \text{ Mev fm}^3, \quad \beta_\infty = \frac{5}{6\pi^2} = 64.8 \text{ MeV fm}^{-3}.$$

- Numerical Mean-field EOS in the asymptotic region

$$\tilde{\alpha} = 938.2 \text{ MeV fm}^3 \quad \text{and} \quad \beta = 61.5\text{Mev fm}^{-3}$$

in agreement with the analytical results within 0.3% and 5% respectively.

E_{026} Model

- Fitting parameters:

$$\tilde{\alpha}_{\epsilon=0.01} = 938.4, \quad \tilde{\alpha}_{\epsilon=0.1} = 939.0, \quad \tilde{\alpha}_{\epsilon=1} = 940.6.$$

- Closer to the BPS limit, we also get the subleading (linear in the volume) term:

$$\beta_{\epsilon=0.01} = 62.2 \text{ MeV fm}^{-3},$$

  with, however, a non-zero value for the off-set.

- It is also possible to fit $\tilde{\beta} V^{1/3} + \beta V$ curve. For $\epsilon = 0.1$ and $\epsilon = 1$ it leads to a negative value for β, which seems to indicate that such a curve is rather not the right one.

E_{0246} Model – I

- New calibration: we fit the parameters of the BPS part of the model to the *mass of the helium nucleon* $m_{He}/4 = 931.75$ MeV and *size of the nucleon* $r_N = 1.25$ fm.
- For the BPS Skyrme model we have

$$E_{BPS} = \frac{2}{12\pi^2} 2\pi^2 \lambda\mu, \quad R = \left(\frac{3\pi}{2}\right)^{1/3} \left(\frac{\lambda}{2\mu}\right)^{1/3}.$$

  Then, $\lambda^2 = 2317$ Mev fm^3, $\mu^2 = 3372$ MeV fm^{-3} in physical units or $\lambda^2 = 8.14$ and $\mu^2 = 2.19$ in the Skyrme units.
- fitting values of the leading term in the energy-mass relation

$$\tilde{\alpha}_{\epsilon=1} = 1929\,\text{MeV fm}^3,\ \tilde{\alpha}_{\epsilon=0.1} = 1908\,\text{MeV fm}^3,$$
$$\tilde{\alpha}_{\epsilon=0.01} = 1905\,\text{MeV fm}^3,$$

  which can be compared with the theoretical value $\tilde{\alpha} = \frac{\lambda^2\pi^2}{12} = 1905$ MeV fm^3.

E_{0246} Model – II

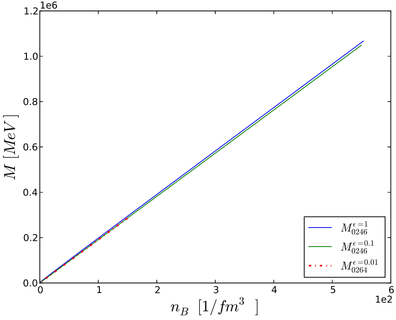
- the sextic term gives the leading behavior for the mass-volume formula in the asymptotic regime.
- the subleading contribution, emerging from the quartic part of the action, has the form $\alpha V^{-1/3}$, with the following fitting values:

$$\alpha_{\epsilon=1} = 924.7\,\text{MeV fm},\ \alpha_{\epsilon=0.1} = 92.7\,\text{MeV fm},\ \alpha_{\epsilon=0.01} = 9.8\,\text{MeV fm},$$

➡ satisfies the relation $\alpha_\epsilon = \epsilon\, \alpha_{\epsilon=1}$.

➡ test for our numerics.

E_{0246} – mass and pressure



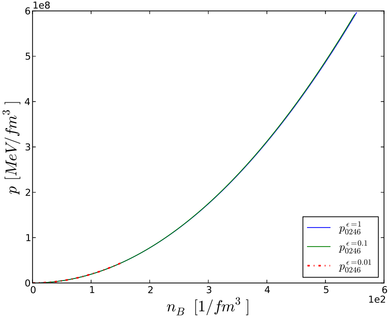


Figure : Mass and pressure as a function of the average baryon density for E_{0246} model

E_{0246} – mass and pressure

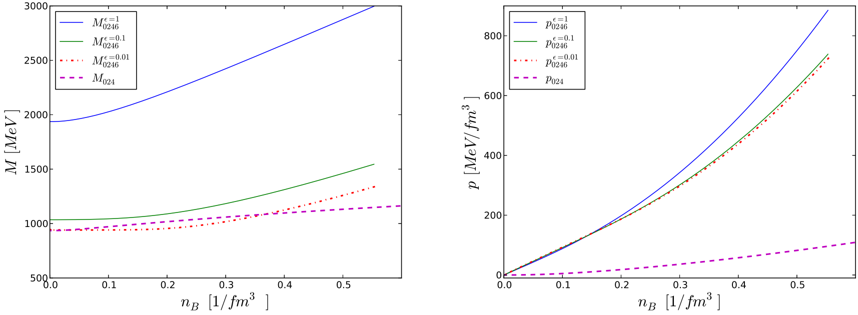


Figure : Mass and pressure as a function of the average baryon density for E_{0246} model

Walecka Model & Conclusions

Walecka Model

$$\boxed{\mathcal{L}_W = \mathcal{L}_N + \mathcal{L}_{\sigma,\omega} + L_{int}},$$

where

$$\mathcal{L}_N = \overline{\psi}\left(i\gamma^\mu\partial_\mu - m + \mu\gamma^0\right)\psi\,,$$

$$\mathcal{L}_{\sigma,\omega} = \frac{1}{2}\left(\partial_\mu\sigma\right)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu\,,$$

$$\mathcal{L}_{int} = g_\sigma\overline{\psi}\sigma\psi + g_\omega\overline{\psi}\gamma^\mu\omega_\mu\psi\,,$$

and $\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$.

Compute partition function $Z = \int \mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{D}\sigma\mathcal{D}\omega\, e^{\int \mathcal{L}_W}$ in the limit where the bosonic field are approximated by their condensate values $\overline{\sigma}, \overline{\omega}_0$.

$$\Rightarrow \mathcal{L}_W = \overline{\psi}\left(i\gamma^\mu\partial_\mu - m^* + \mu^*\gamma^0\right)\psi - \tfrac{1}{2}m_\sigma^2\overline{\sigma}^2 + \tfrac{1}{2}m_\omega^2\overline{\omega}_0^2$$

where

$$m^* = m - g_\sigma\overline{\sigma}\,,\quad \mu^* = \mu - g_\omega\overline{\omega}_0$$

Walecka Model

- Pressure and energy density

$$P = \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\bar{\rho}_B^2 - \frac{1}{2}\frac{g_\sigma^2}{m_\sigma^2}n_s^2 + \frac{1}{4\pi^2}\left[\left(\frac{2}{3}k_F^3 - (m^*)^2 k_F\right)E_F^* \right.$$
$$\left. + (m^*)^4 \ln\frac{k_F + E_F^*}{m^*}\right],$$
$$\bar{\varepsilon} = \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\bar{\rho}_B^2 + \frac{1}{2}\frac{g_\sigma^2}{m_\sigma^2}n_s^2 + \frac{1}{4\pi^2}\left[\left(2k_F^3 + (m^*)^2 k_F\right)E_F^* \right.$$
$$\left. - (m^*)^4 \ln\frac{k_F + E_F^*}{m^*}\right],$$

- Fermi energy and nucleon mass

$$E_F^* = \sqrt{k_F^2 + (m^*)^2}, \quad m^* = m - \frac{g_\sigma^2}{m_\sigma^2}n_s.$$

- baryon and scalar densities at $T = 0$

$$\bar{\rho}_B = \frac{2k_F^3}{3\pi^2}, \quad n_s = \frac{m^*}{\pi^2}\left[k_F E_F^* - (m^*)^2 \ln\frac{k_F + E_F^*}{m^*}\right].$$

Walecka Model

- leading behaviour of the mean-field energy density

$$\bar{\varepsilon} = \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\bar{\rho}_B^2 + \frac{3}{4}\left(\frac{3\pi^2}{2}\right)^{1/3}\bar{\rho}_B^{4/3} + \frac{1}{2}\frac{m_N^2 m_\sigma^2}{g_\sigma^2},$$

We observe:

1. The asymptotic formulae for the equations of state in the full (i.e., containing the BPS part) Skyrme model and in the Walecka model coincide.
2. Coincidence also between the first subleading terms, which behave as $\bar{\rho}_B^{4/3}$.
3. Constant term
4. There are no further terms in the large density limit of the Walecka model which could correspond to the term generated by the sigma model part of the Skyrme model. Namely, a term behaving as $\bar{\rho}^a$ with $4/3 > a > 0$.

Conclusions& Future Work

Conclusions

1. New insight into the MF-EoS in general Skyrme models
2. Relation between Skyrme and Walecka model

Future Work

1. What is the MF-EoS in the limit of infinite nuclear matter?
2. What is the energy minimizer for a given B on a given compact manifold with a certain volume?
3. What happens for nonzero Temperature $T > 0$?

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