Rigid Body Motion in a Special Lorentz Gas

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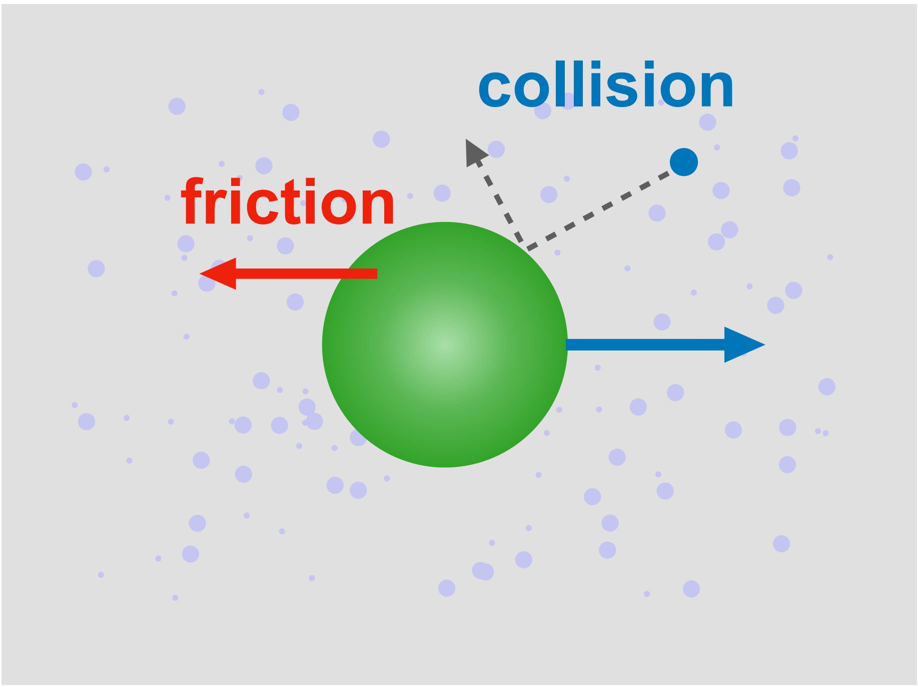
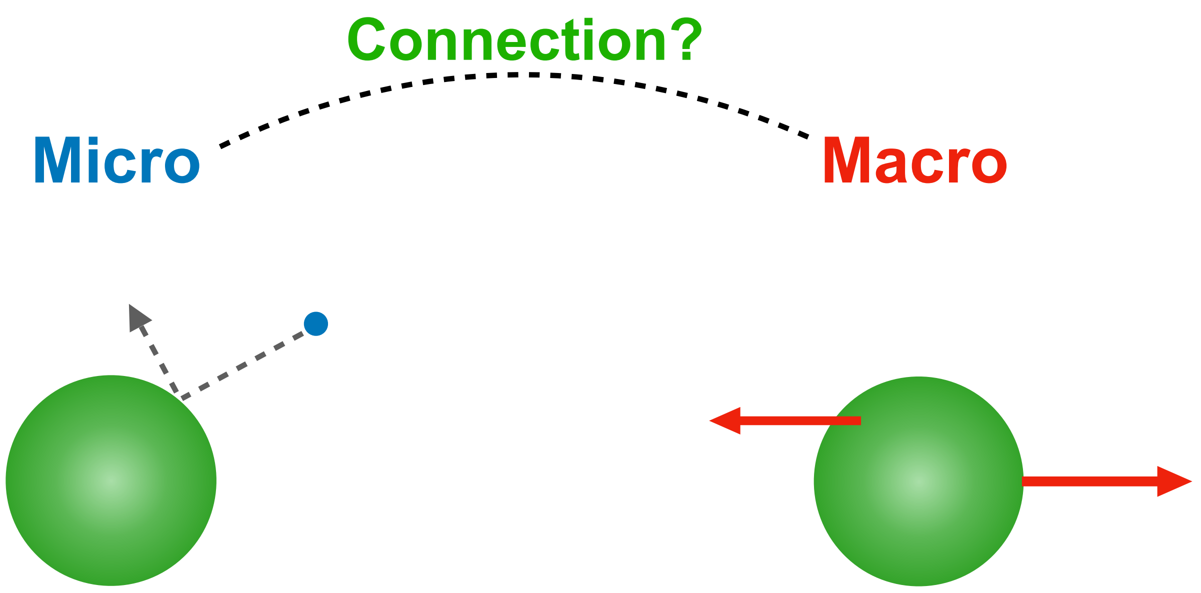
Introduction

Microscopic origin of friction

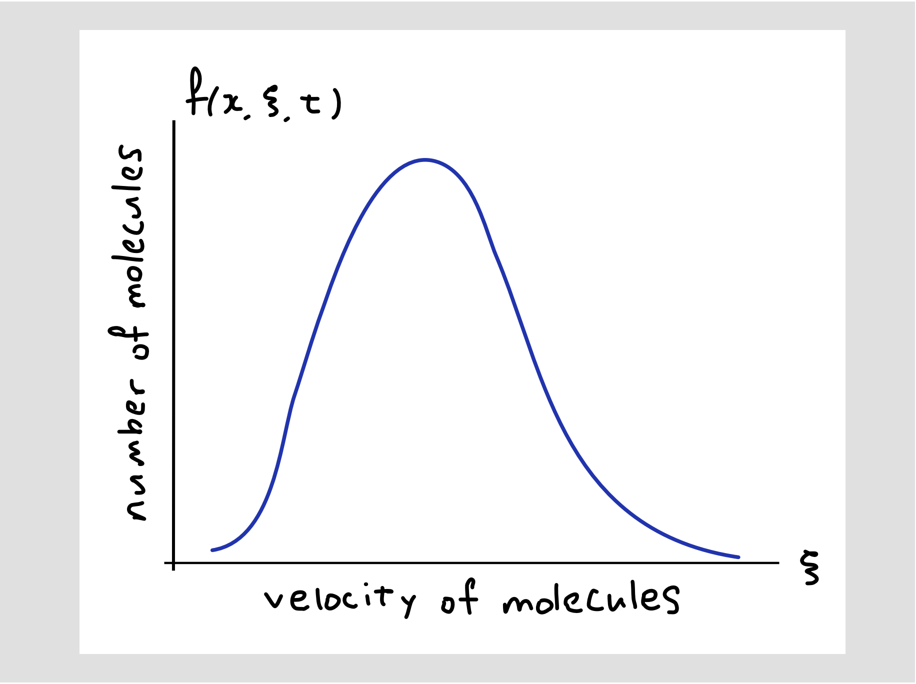




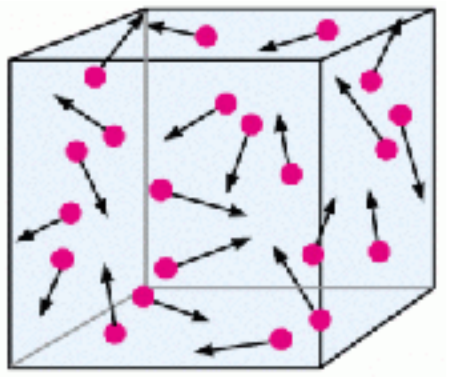
☆ Micro and macro dynamics are strongly connected

Microscopic origin of friction

Kinetic theory of gases



Velocity distribution function: $f(x, \xi, t)$



Δx

$$\int_{\Delta x \times \Delta \xi} f \, dx d\xi$$

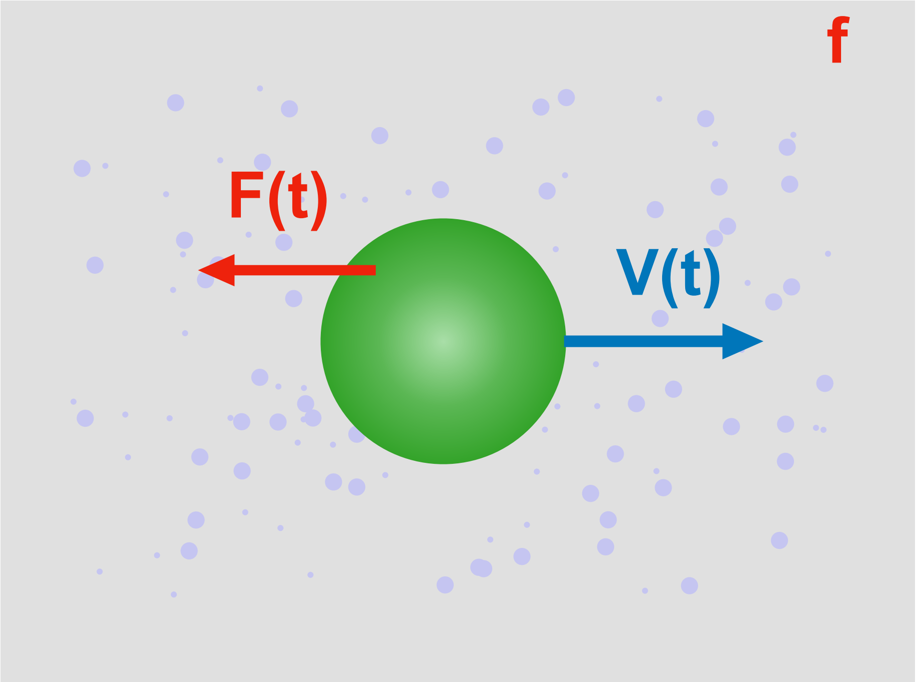
average number of molecules

Macroscopic quantities

Density $\rho(x,t) = \int f(x,\xi,t)\, d\xi$

Mean velocity $u(x,t) = \int \xi f(x,\xi,t)\, d\xi / \rho(x,t)$

Mathematical questions: Long time behavior of a rigid body



	Gas		Moving body
	f		$V(t)$
PDE	$\partial_t f + \xi \cdot \nabla_x f = I(f)$	**ODE**	$\dfrac{dV(t)}{dt} = -F(t)$
BC	$f(x,\xi,t) = f(x,\xi'_{V(t)},t)$		$F(t) = \int_S \int_\xi \cdots f \, d\xi dS$

1. Can we solve two different equations simultaneously?
2. What is the decay rate of V(t)?
3. Micro-macro connection?

- Previous results: I(f)=0
- Main result: an example of I(f)≠0

Rigid body motion in a special Lorentz gas

Special Lorentz gas **[Tsuji & Aoki (2012)]**

$$\partial_t f + \xi \cdot \nabla_x f = \nu_\varepsilon(|\xi|)(f_0 - f)$$

$$\begin{cases} \varepsilon + C^{-1}\dfrac{z^2}{z+\varepsilon} \leq \nu_\varepsilon(z) \leq \varepsilon + C\dfrac{z^2}{z+\varepsilon} \\ f_0(\xi) = \pi^{-3/2}\exp(-|\xi|^2) \end{cases}$$

<u>**Theorem**</u> **[K., J. Stat. Phys. (2018)]**

For the special Lorentz gas, the decay rate of V(t) is

- $\varepsilon > 0$: exponential $V(t) \approx e^{-\varepsilon t}$
- $\varepsilon = 0$: algebraic $V(t) \approx t^{-5}$

Mathematical Formulation

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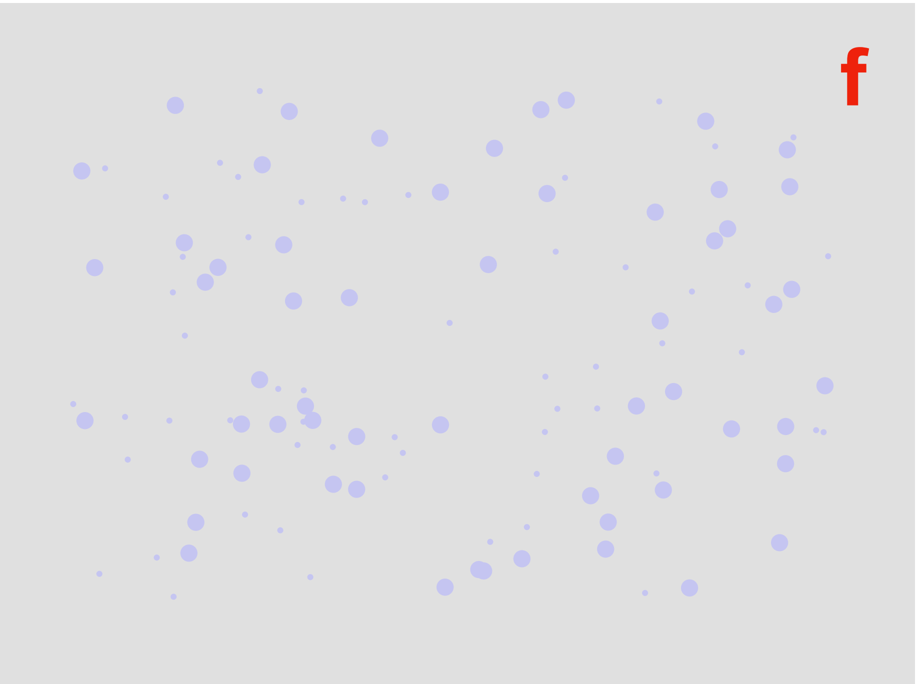
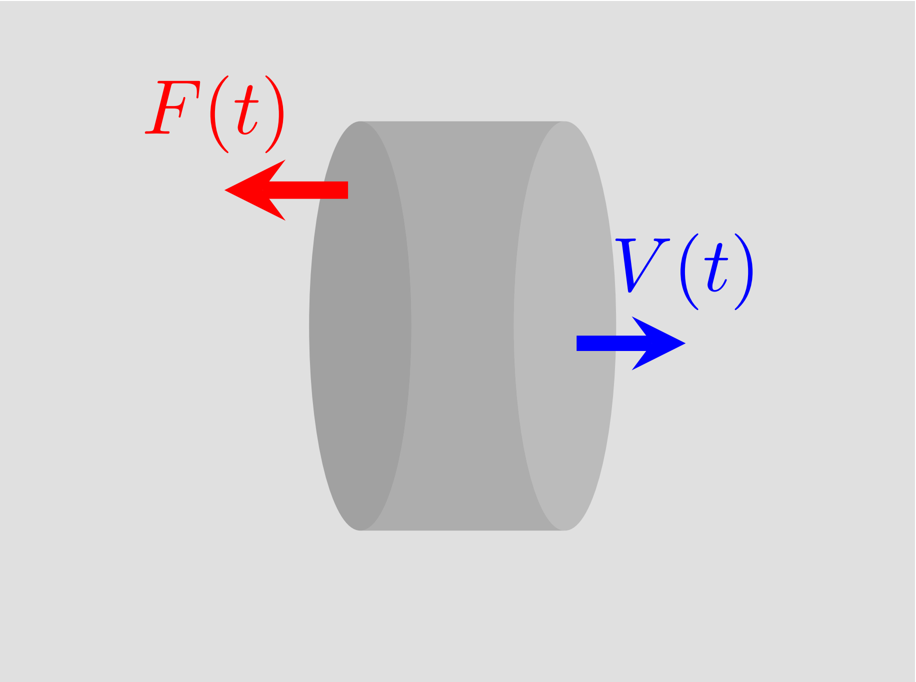


Kinetic equation

$$\partial_t f + \xi \cdot \nabla_x f = I(f)$$

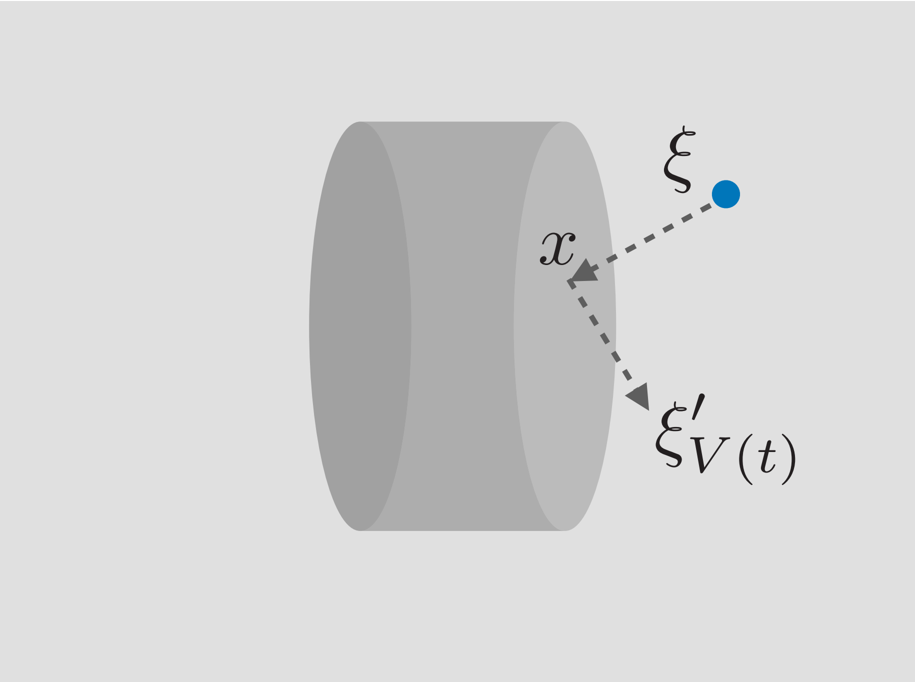
$I(f)$: interaction term



Newton's equation

$$\frac{dV(t)}{dt} = -F(t)$$

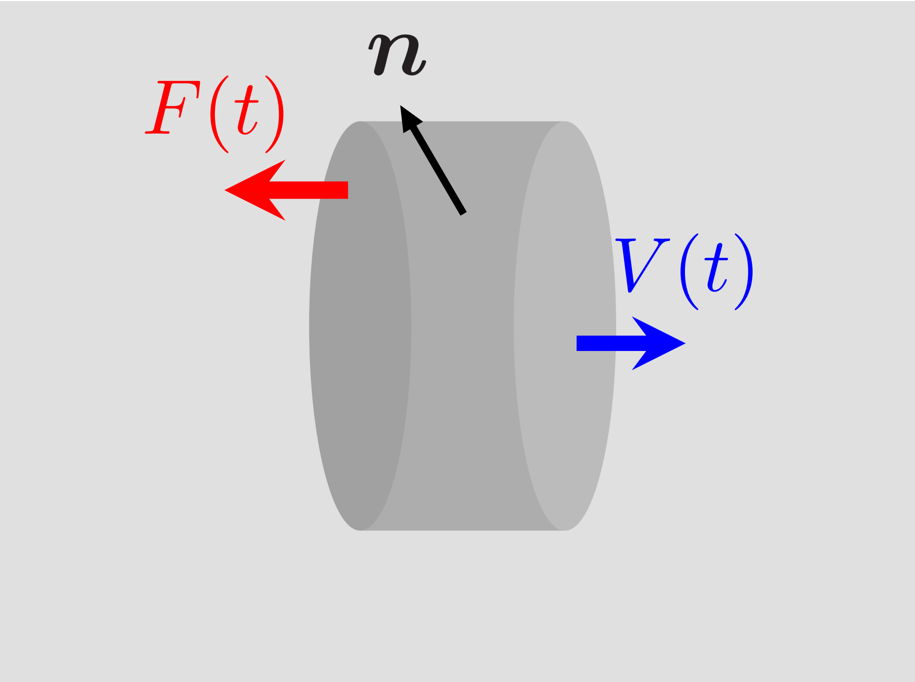
Mathematical Formulation



Boundary condition

$$f(x,\xi,t) = f(x,\xi'_{V(t)},t)$$

$$(\xi'_{V(t)} - V(t)\boldsymbol{e}_1)\cdot\boldsymbol{n} = -(\xi - V(t)\boldsymbol{e}_1)\cdot\boldsymbol{n}$$



Friction

$$F(t) = \int_S \int_\xi \xi_1(\xi - V(t)\boldsymbol{e}_1)\cdot\boldsymbol{n} f \, d\xi dS$$

Mathematical Formulation

Gas	Moving body
f	$V(t)$
$\partial_t f + \xi \cdot \nabla_x f = I(f)$	$\dfrac{dV(t)}{dt} = -F(t)$
$f(x, \xi, t) = f(x, \xi'_{V(t)}, t)$	$F(t) = \int_S \int_\xi \cdots f \, d\xi dS$

Coupled in both ways

Previous Result

Decay rate of V(t): Previous result

Free molecular flow: $\partial_t f + \xi \cdot \nabla_x f = 0$

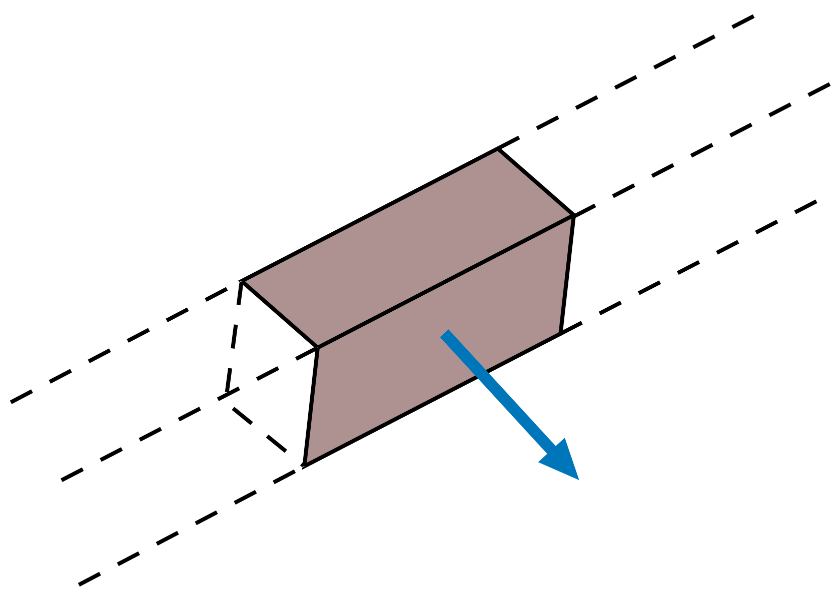
Theorem [Caprino et. al., CMP (2007)]

For free molecular flow, V(t) decays algebraically as

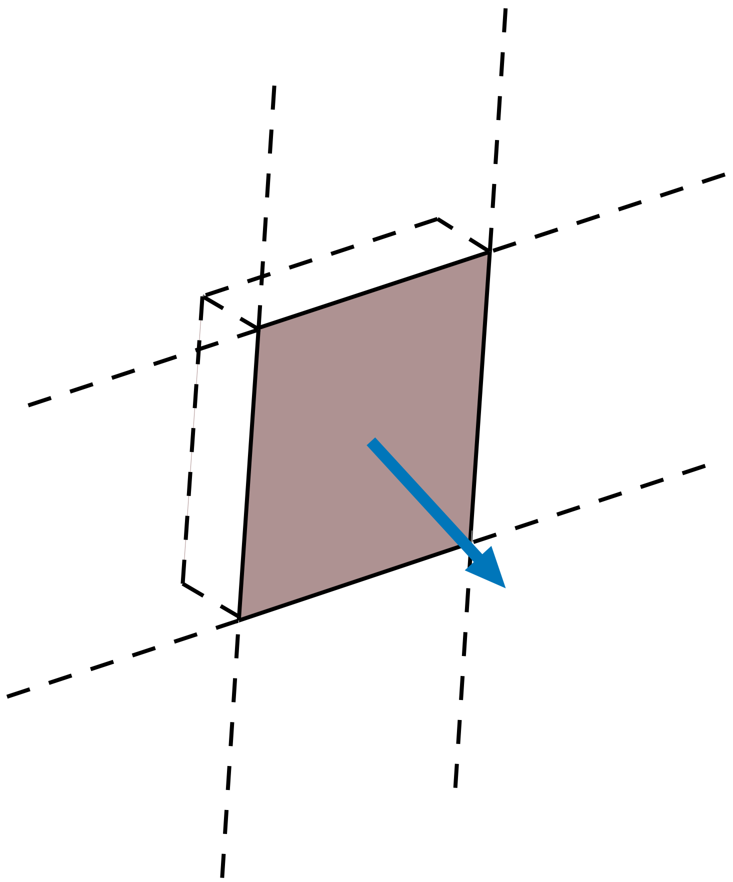
- $V(t) \approx t^{-(d+2)}$

One and two dimensional cylinders

- $V(t) \approx t^{-(d+2)}$



d=2: Infinite rod



d=1: Infinite wall

Decay rate of V(t): Previous result

Free molecular flow: $\partial_t f + \xi \cdot \nabla_x f = 0$

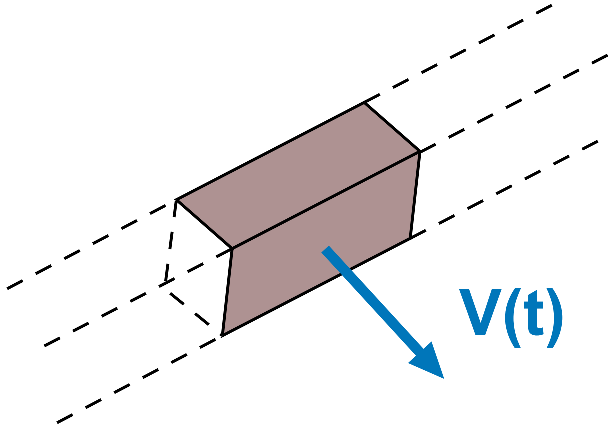
Theorem **(Caprino et. al., 2007)**

For free molecular flow, V(t) decays algebraically as

- $V(t) \approx t^{-(d+2)}$

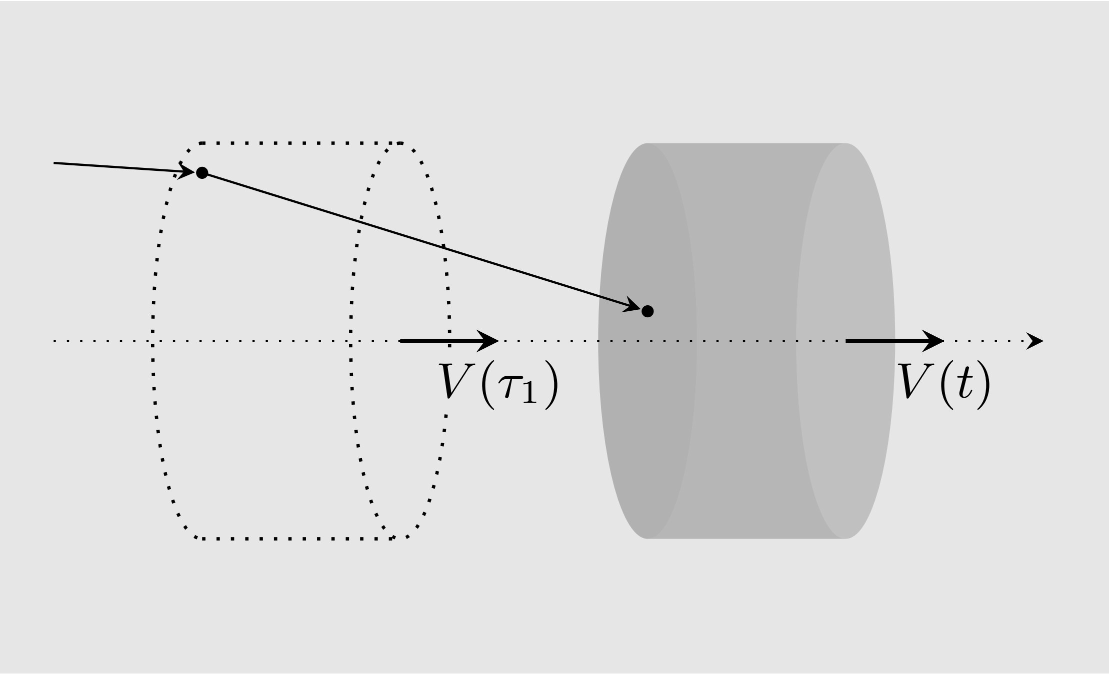
- $F(t) = CV(t) \Rightarrow V(t) = V_0 e^{-Ct}$

Why algebraic?



d=2: Infinite rod

Recollision



$F(t) = CV(t)$	$F(t) = \int\int \cdots f \, d\xi dS$
Markov	**Non-Markov**
Exponential $V(t) \approx e^{-Ct}$	**Algebraic** $V(t) \approx t^{-(d+2)}$

Multiple collision of the molecules

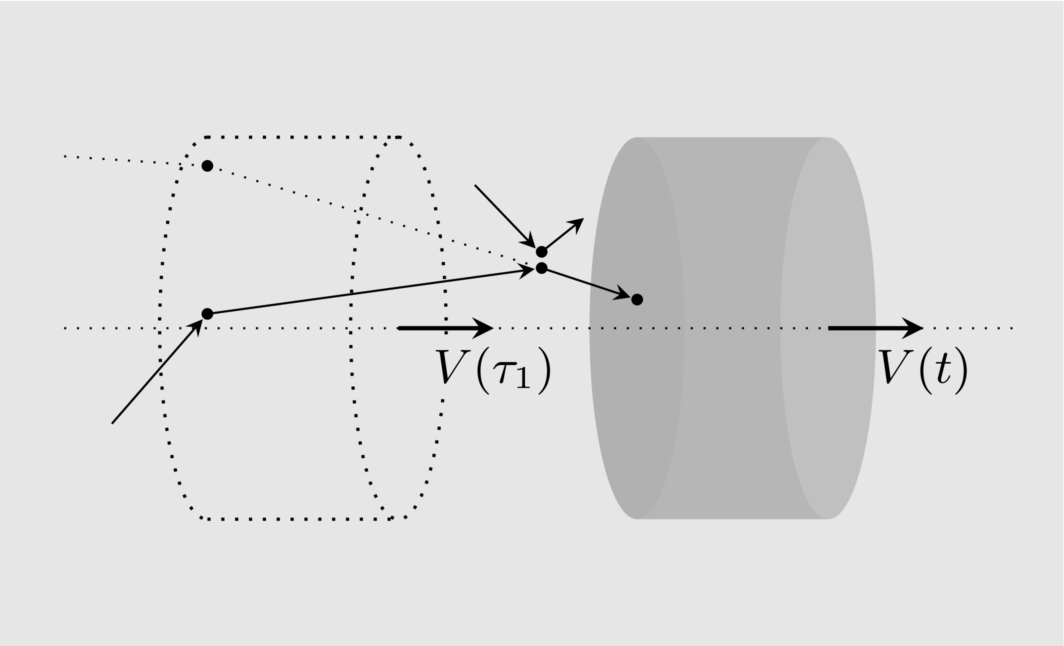
Micro-scale: recollision

F(t) depends on V(s) (0<s<t)

Macro-scale: non-Markov

Main Result

Question: Interaction destroys non-Markovness?



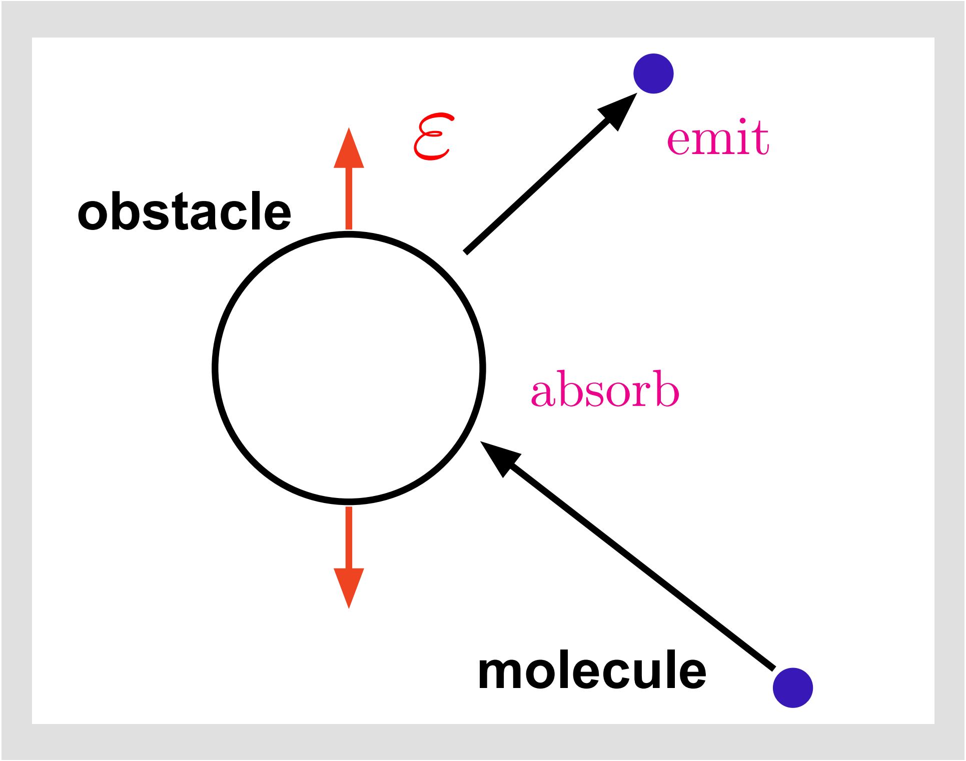
- **Scattering destroys information of the past**
- **Exponential decay?**
- Numerical simulation

[Tsuji & Aoki (2012)]

Analyzed a special Lorentz gas

Can we have a mathematical theory?

Interaction with obstacles (condensed phase)



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ε: obstacle's average speed

Decay rate of V(t): Special Lorentz gas

<u>**Theorem**</u> **[K., J. Stat. Phys. (2018)]**

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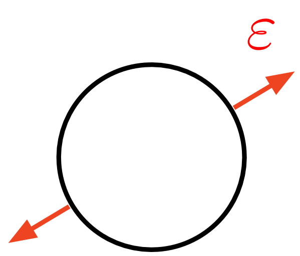


- Confirms the numerical simulation

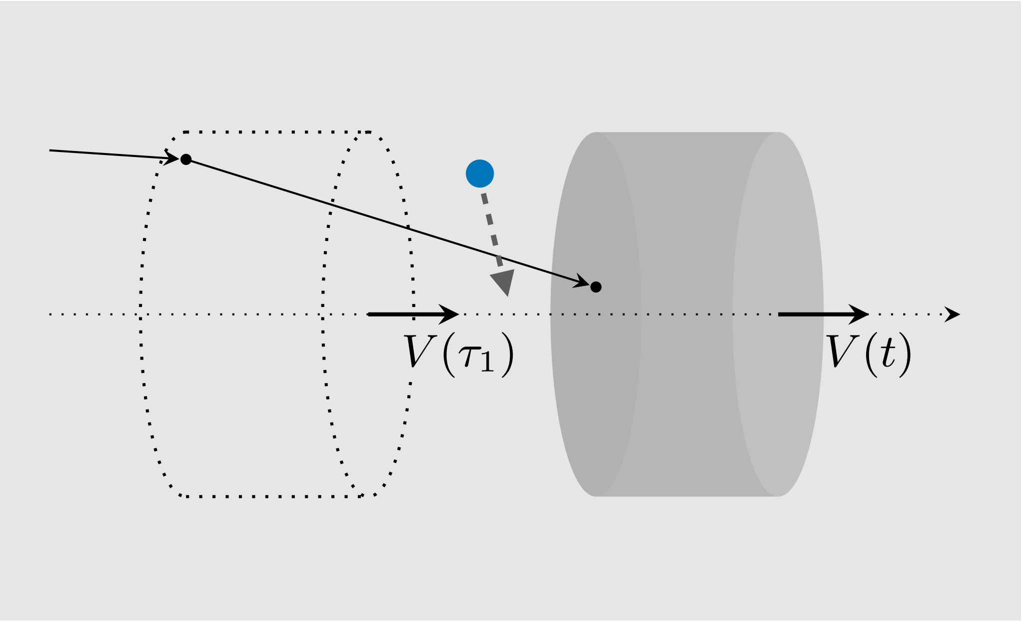
[Tsuji & Aoki (2012)]

- Decay rate is independent of d

Intuitive explanation?

Dependence on ε

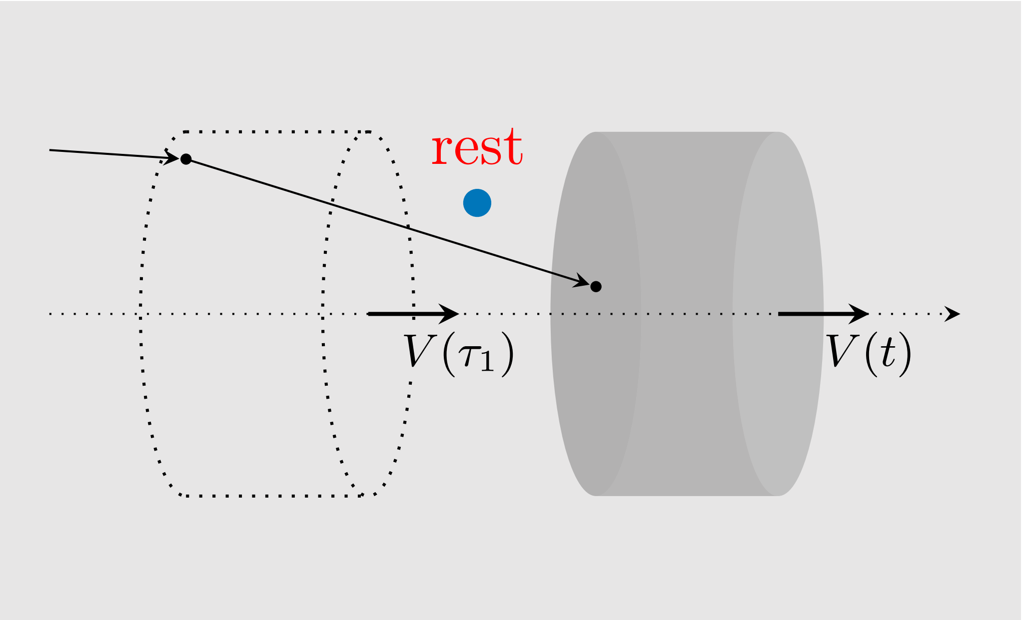
$\varepsilon > 0$



Molecules are scattered most of the time

Almost Markov; exponential decay

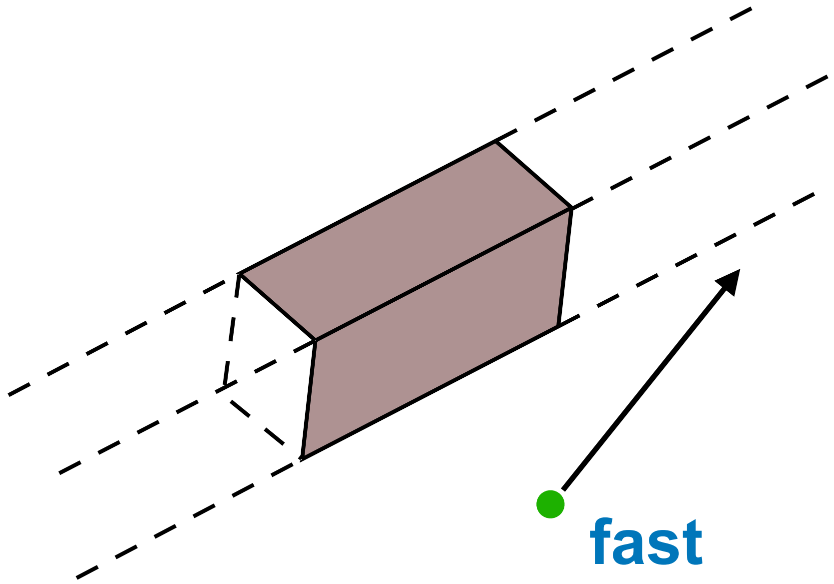
$\varepsilon = 0$

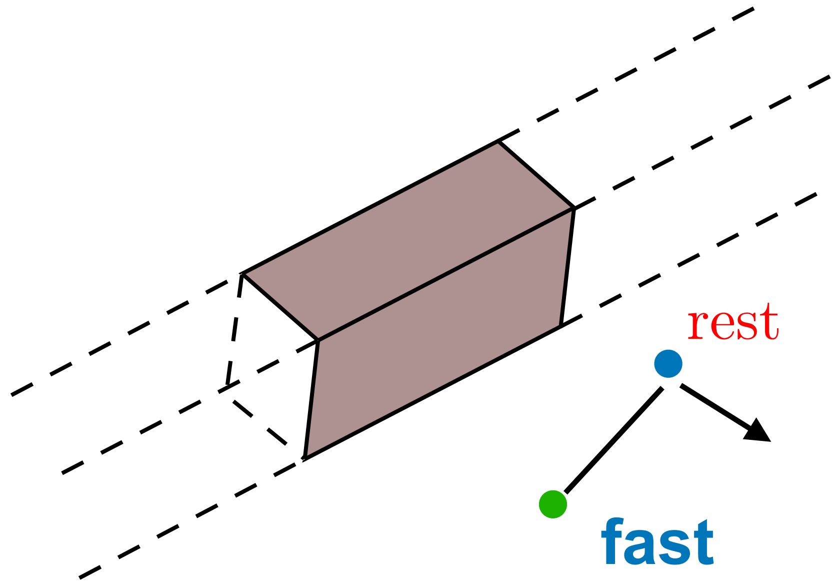


Slow molecules are sometimes not scattered

Weakly non-Markov; inverse power decay

Independence on d





Decay rate of V(t): Special Lorentz gas

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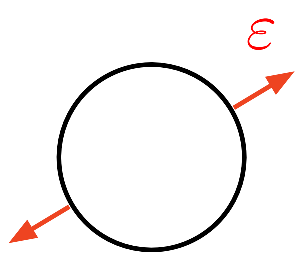

- Confirms the numerical simulation

[Tsuji & Aoki (2012)]

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Intuitive explanation?

Conclusion

Conclusion

Summary

1. Micro-macro dynamics are strongly connected — recollision

2. Interaction destroys non-Markovness

Further question:

What about the Boltzmann equation? (Really hard question)

Visit my webpage for the paper and the slide:
Google as “Kai Koike”