Decorated enhanced Teichmüller space

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- Main result

2 Preparation

$\hat{S}$: conn. cpt. ori. surf. which has genus $g \geq 0$,

$c \geq 0$ boundary components,

$p \geq 0$ internal points, and

$s \geq 0$ boundary points.

$P := \{\text{internal point}\}$,

$V := \{\text{internal and boundary point}\}$

$S := \hat{S} \setminus V$

Assume $4 - 4g - 2c - 2p - s < 0$ and that each boundary has at least 1 boundary point.

3 Teichmüller space

X : cplt finite area hyperbolic surf. which has cusps, closed geodesic boundaries, crowns.

Marking $f : S \to X \setminus \{\mathrm{c.g.b.}\}$ ori-pre. homeo.

$(X_1, f_1) \sim (X_2, f_2) \Leftrightarrow$
${}^{\exists}\varphi : X_1 \to X_2$ ori-pre. isom. $\varphi \circ f_1 \sim f_2$

Teichmüller sp. $\mathcal{T}(S) :=$
$\{\text{marked hyp. surf. (without c.g.b.)}\}/\sim$

4 Enhanced Teichmüller space

Enhancing $\varepsilon : \{\text{c.g.b. of } X\} \to \{\pm 1\}$

$(X_1, f_1, \varepsilon_1) \sim (X_2, f_2, \varepsilon_2) \Leftrightarrow$
$(X_1, f_1) \sim (X_2, f_2)$ and $\varepsilon_2 \circ \varphi = \varepsilon_1$ on $\{\text{c.g.b.}\}$

Enhanced Teichmüller sp.
$\mathcal{T}^x(S) := \{\text{enhanced marked hyp. surf.}\}/\sim$

5 Shearing coordinates

Δ : triangulation of $\hat{S}$

Shearing of edge α
Signed length of the gluing gap at α

Prop. $\mathcal{T}^x(S)$ is homeo. to $\mathbb{R}^{\mathrm{int}E(\Delta)}$ by measuring shearing of internal edges, where $\mathrm{int}E(\Delta)$ is the set of internal edges of Δ.

6 Decorated Teichmüller space

Decoration curve : horocycle at cusp and
horocyclic arc at spike

Decoration $D := \bigcup_{v \in V} \{\text{decoration curve at } v\}$

$(X_1, f_1, D_1) \sim (X_2, f_2, D_2) \Leftrightarrow$
$(X_1, f_1) \sim (X_2, f_2)$ and $\varphi(D_1) = D_2$

Decorated Teichmüller sp. $\mathcal{T}^a(S) :=$
$\{\text{decorated marked hyp. surf. without c.g.b.}\}/\sim$

7 λ-length coordinates

λ-length of α

Signed length along α between decoration curves

Prop. $\mathcal{T}^a(S)$ is homeo. to $\mathbb{R}^{E(\Delta)}$ by measuring λ-length of edges, where $E(\Delta)$ is the set of edges of Δ.

8 Deco. Enha. Teichmüller sp.

Decoration curve at c.g.b. : equidistant curve

$(X_1, f_1, \varepsilon_1, D_1) \sim (X_2, f_2, \varepsilon_2, D_2) \Leftrightarrow$
$(X_1, f_1) \sim (X_2, f_2)$, $\varepsilon_2 \circ \varphi = \varepsilon_1$ on $\{\mathrm{c.g.b.}\}$
and $\varphi(D_1) = D_2$

Decorated enhanced Teichmüller sp.
$\mathcal{T}^{xa}(S) := \{\text{deco. enha. marked hyp. surf.}\}/\sim$

9 Shearing, decoration coord.

Decoration parameter

d is $\mathbb{R}$-valued parameter of boundary length :

- $d(r) = 0 \Leftrightarrow r$ is length of collar boundary,
- $\frac{\mathrm{d}d}{\mathrm{d}r} = \frac{\mathrm{d}\lambda}{\mathrm{d}r}$.

Assume $s > 0$.

Thm. $\mathcal{T}^{xa}(S)$ is homeo. to $\mathbb{R}^{\mathrm{int}E(\Delta)} \times \mathbb{R}^V$ by measuring shearing of internal edges and decoration parameter at point of V.

10 λ, boundary length coord.

Thm. $\mathcal{T}^{xa}(S)$ is homeo. to $\mathbb{R}^{E(\Delta)} \times \mathbb{R}^{P}$ by measuring λ-length of edges and boundary length at points of P.

Idea of proof

- Homeo. $\mathbb{R}^{\mathrm{int}E(\Delta)} \times \mathbb{R}^{V} \to \mathbb{R}^{E(\Delta)} \times \mathbb{R}^{P}$
- Calculation at special triangulation Δ_0
- Flip deformation from Δ_0 to Δ
- Extended Ptolemy equation

11 Future work

- Case of $s = 0$
- Extend the Poisson structure on $\mathcal{T}^x(S)$
- Extend the degenerate symplectic structure on $\mathcal{T}^a(S)$

Thank you for your attention !