SYLLABUS FOR MA531, MATHEMATICAL LOGIC

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Course Description

The is a high-level course in mathematical logic, the field at the intersection of mathematics and logic that investigates logical reasoning with mathematical methods. A rigorous treatment of the formal syntax of first-order logic lays the basis. Then the formal semantics according to Tarski's definition of truth is given. A formal theory of deduction is subsequently developed. Then the fundamental Gödel Completeness Theorem is established, coordinating semantics and deduction, meaning and consequence. After pursuing some of the corollaries like the Compactness Theorem, Gödel's Incompleteness Theorem is sketched at the end, and its ramifications for mathematics and philosophy considered.

This course is intended for students interested in the investigation of reasoning itself, as construed in terms of formal languages and structures. There should be considerable background in mathematics, if only to assimilate the approach through mathematical notation and theorems. Mathematics experienced by having taken two years of calculus and a year of modern mathematics (discrete mathematics, abstract algebra) would be especially helpful. Having taken critical thinking and logic courses in philosophy would also be helpful, for raising awareness of the kind of issues involved.

BU Hub Learning Outcome

Philosophical Inquiry and Life's Meanings, a unit of Philosophical, Aesthetic, and Historical Interpretation. Logic, as first brought to the fore by Aristotle, is at the center of philosophical inquiry; how we reason and the modes of reasoning are at the heart of our sense of the rational. Frege in the 19th Century incorporated the structure of quantification into logic, and this stimulated the modern development of mathematical logic—logic articulated, studied, and analyzed in mathematical terms. It is prominently through mathematical structures and argumentation that logic, particularly the investigation of syntax and semantics, proceeds today with the philosophical enquiry into how we think and make deductions.

Outcomes for Majors and Minors

This course fulfills upper-level requirements for the following majors: Mathematics, Mathematics and Philosophy, Philosophy. For the latter two, one can take the course as PH461.

Required Text

Herbert Enderton, *A Mathematical Introduction to Logic,* Second Edition, Harcourt/ Academic Press 1977. Available at the BU Bookstore. On course reserve at the Science and Engineering Library.

Grading and Procedures

The course grading is broken down into 8 units as follows: Exercises, 4; Final, 2; Midterm, 1; Attendance, 1. The percentage-wise worst unit is dropped, and the rest are given equal weight in calculating the grade. For example, suppose that your Exercise grade (points achieved over points possible for all assigned exercises) was .91; your Final was .81; your Midterm was .85, and your attendance was .90. Then one unit of the Final will be dropped, and your cumulation will therefore be: .91 x 4 + .81 x 1 + .85 x 1 + .90 x 1.

Attendance is calculated according to your signing an attendance sheet circulated at each class meeting.

The *Midterm* will be administered in the week after the semester break.

The *Final* will be administered at the mandated time according to the BU calendar.

Exercises will be periodically assigned, collected on the date due, and returned graded; they can be recycled once for full credit up to the collection date of the succeeding assignment.

Topics and Exercises

The following topics, and corresponding exercises to be assigned, are drawn from the text. The coverage and pace with which we will proceed in class depends in part on class participation and the preparedness of the students. In particular, some exercises may be dropped or altered, and some supplementary exercises may be assigned. Each week the exercises due will be specified according to section.

The Language of Sentential Logic. 1.1: 2,3,5.
Truth Assignments. 1.2: 1,3,4,5,7,8,9,12,13.
Sentential Connectives. 1.5: 1,9.
Compactness and Effectiveness. 1.7: 1,2,3.
First-Order Languages. 2.1: 1–8.
Truth and Models. 2.2: 2,3,4,6,8,9,11.
A Deductive Calculus. 2.4: 2,6,7,8,10,15,17.
Soundness and Completeness Theorems. 2.5: 2,4,6,7.
Models of Theories. 2.6: 2,3,8.

Other Class and University Policies

Absences from class will be noted according to attendance sheets as mentioned above; the effect of the number of absences will be as in the grading described above. Other times for midterm and final may possibly be arranged, e.g. for religious observances, beforehand.

Accommodations for Students with Documented Disabilitiies will be made by arrangement; this typically may involve extra time on exams.

The Incomplete Grade I is given only in very exceptional cases to students who have maintained a good record through much of the course and suddenly find themselves in very difficult circumstances. A definite arrangement will then be made for clearing the incomplete grade. Others who find early on that they are not keeping up are urged to drop or withdraw from the course.

Academic Conduct is according to: https://www.bu.edu/academic/policies/academic-conduct code. Cheating or plagiarism on exams is not tolerated, and will be handled according to the code.