SYLLABUS FOR MA532, FOUNDATIONS OF MATHEMATICS

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Course Description

The is a high-level course that develops axiomatic set theory both as a foundations of mathematics and a field of mathematics that investigates the infinite. The course begins with a review of informal set theory and first-order logic. Proceeding through the basic Zermelo axioms and operations, relations and functions are incorporated and mathematical concepts of number from integers to reals reals are objectified. Then Cantor's transfinite numbers, cardinal and ordinal, are developed. Proceeding according to the historical unfolding, we conclude with Cantor's Continuum Hypothesis and Zermelo's Axiom of Choice as conduits to modern set theory.

This course is intended for students interested in seeing a unifying foundations of mathematics, working through an ultimate concept calculus that features a coherent account of the infinite, and assimilating a modern field of mathematics presented logically and axiomatically from scratch. Students should have some understanding of first-order logic, and generally the sort of mathematical experience acquired by having taken two years of calculus and a year of modern mathematics (discrete mathematics, abstract algebra), both to be able to proceed with the axiomatic presentation and to appreciate the issues involved.

BU Hub Learning Outcome

Philosophical Inquiry and Life's Meanings, a unit of Philosophical, Aesthetic, and Historical Interpretation. At the heart of philosophical inquiry, since Plato and Aristotle, is epistemology (knowledge) and ontology (existence). From the beginning, mathematics was the field of play for inquiry in these directions, and as mathematics developed through millennia that inquiry became more and more articulated as mathematics found more and more application in the external world. Coming full circle at the beginning of the 20th Century, epistemology and ontology became themselves part of the incentives of mathematical inquiry, now into its possible, logical foundations as a coherent science. Emerging out of this was axiomatic set theory, which directly addresses what and how we (get to) know about concepts and their interaction—particularly about the infinite—and what we can take to exist in scientific context. Thus, axiomatic set theory and the foundations of mathematics provide the modern, structured setting for the further inquiry into epistemology and ontology.

Outcomes for Majors and Minors

This course fulfills upper-level requirements for the following majors: Mathematics, Mathematics and Philosophy, Philosophy. For the latter two, one can take the course as PH462.

Required Text

Herbert Enderton, *Elements of Set Theory* , Academic Press 1977. Available at the BU Bookstore. On course reserve at the Science and Engineering Library.

Grading and Procedures

The course grading is broken down into 8 units as follows: Exercises, 4; Final, 2; Midterm, 1; Attendance, 1. The percentage-wise worst unit is dropped, and the rest are given equal weight in calculating the grade. For example, suppose that your Exercise grade (points achieved over points possible for all assigned exercises) was .91; your Final was .81; your Midterm was .85, and your attendance was .90. Then one unit of the Final will be dropped, and your cumulation will therefore be: .91 x 4 + .81 x 1 + .85 x 1 + .90 x 1.

Attendance is calculated according to your signing an attendance sheet circulated at each class meeting.

The *Midterm* will be administered in the week after the semester break.

The *Final* will be administered at the mandated time according to the BU calendar.

Exercises will be periodically assigned, collected on the date due, and returned graded; they can be recycled once for full credit up to the collection date of the succeeding assignment.

Topics and Exercises

The following topics, and corresponding exercises to be assigned, are drawn from the text. The coverage and pace with which we will proceed in class depends in part on class participation and the preparedness of the students. In particular, some exercises may be dropped or altered, and some supplementary exercises may be assigned. Each week the exercises due will be specified according to section.

Chapter 2: Axioms and Operations. Arbitrary Unions and Intersections, 2-10; Algebra of Sets, 11-14,23,24.

Chapter 3: Relations and Functions. Ordered Pairs, 1-5; Relations, 6-9; Functions, 15,16,19,22,28,29; Equivalence Relations, 32-35; Ordering Relations, 43-45.

Chapter 4: Natural Numbers. Peano's Postulates, 2-6; Arithmetic, 13-17; Ordering on ω, 20-23,26.

Chapter 6: Cardinal Numbers and the Axiom of Choice. Equinumerosity, 4,5; Finite Sets, 6,7; Cardinal Arithmetic, 10,11; Axiom of Choice, 19,20; Arithmetic of Infinite Cardinals, 32-34.

Chapter 7: Orderings and Ordinals. Partial Orderings, 1,2; Well Orderings, 5-7; Replacement Axioms, 8,9; Epsilon Images, 10,11; Ordinal Numbers, 15-18; Debts Paid, 23-25.

Other Class and University Policies

Absences from class will be noted according to attendance sheets as mentioned above; the effect of the number of absences will be as in the grading described above. Other times for midterm and final may possibly be arranged, e.g. for religious observances, beforehand.

Accommodations for Students with Documented Disabilitiies will be made by arrangement; this typically may involve extra time on exams.

The Incomplete Grade I is given only in very exceptional cases to students who have maintained a good record through much of the course and suddenly find themselves in very difficult circumstances. A definite arrangement will then be made for clearing the incomplete grade. Others who find early on that they are not keeping up are urged to drop or withdraw from the course.

Academic Conduct is according to: https://www.bu.edu/academic/policies/academic-conduct code. Cheating or plagiarism on exams is not tolerated, and will be handled according to the code.