

A PROPOSAL FOR DECIDING CHESS MATCHES

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The title of World Champion of Chess has been universally acknowledged for a century, but rather remarkably, there has not been a synthetic process toward any universally accepted rules for deciding world championship matches. It was just over a decade ago that World Champion Robert J. Fischer renounced his title over a dispute with F.I.D.E. (Fédération Internationale des Echecs, the world chess federation) over the terms of his first title defense, against Challenger Anatoly Karpov. With the recent abrupt and inconclusive termination of the marathon title match in Moscow between Karpov and challenger Gary Kasparov, the subject has once again become topical. What we propose here is a new method for deciding world championship matches, based on a simple mathematical idea; this method can be applied to any chess match, or indeed, to any match of a similar "two-person" game.

In the distant past, the title of World Champion was entirely proprietary to the holder, and he negotiated his own terms for title matches. In the Steinitz-Zukertort match of 1886, which was touted as the first official world championship match and which incidently was the first match using clocks, the terms were that the first player to win 10 games wins the match. If both players at some point have each won 9 games, the match would then be terminated with the champion retaining the title - the outcome of the match should not depend on a single win, namely the tenth. For the next 50 years, almost all title matches followed this precedent of sorts, by specifying some minimum number of wins required to take the match, and often incorporating some provision for a drawn match.

It was during the reign of Alexander Alekhine that a feature which had only occurred sporadically in matches became prominent: a limit on the total number of games to be played. Both his matches against Bogolyubov (1929 and 1934) and against Euwe (1935 and 1937) were decided on the basis of the best score in 30 games, although the winner did have to chalk up at least six wins.

With the sudden death of Alekhine in 1946, F.I.D.E. took advantage of the interregnum by bringing the title of World Champion under its auspices. Title matches, and indeed the procedure for determining challengers, became governed by F.I.D.E. regulations, determined and amended by its periodic congresses. The terms, for title matches at least, were completely stabilized in the next quarter of a century of Soviet hegemony, through the reigns of the Soviet champions Botvinnik, Smyslov, Tal, Petrosian, and Spassky. One simply required the best score out of 24 games, with the champion retaining the title in case of the score 12-12. There was no longer any requirement about actually having to win any games. Also, the apparently remote possibility of a 12-12 drawn match actually occurred in the first two F.I.D.E. matches, Botvinnik-Bronstein (1951) and Botvinnik-Smyslov (1954).

In the early 1970's, Fischer started to raise objections against the F.I.D.E. rules, opining that they tend to deaden title matches with many draws as a natural part of long-term strategy. Technically after all, the champion simply has to draw all his games to retain the title. Fischer himself exemplified the drawing strategy in his epic 1972 match against Champion Boris Spassky by serenely using solid but unassuming variations to coast through the last half of the match with draws, after having buttressed himself with a comfortable lead at the beginning.

Fischer wanted to revert to the minimum number of wins formula, with no limit on the total number of games. In other words, draws should not count, as before Alekhine. For his first title defense, against Karpov in 1975, Fischer eventually got F.I.D.E. to agree to his terms of 10 games necessary to win, with no limit on the number of games. As a student of history and in particular an admirer of Steinitz, he probably had Steinitz-Zukertort and other matches of the period in mind, for it was his further insistence on the fateful 9-9 draw clause that the F.I.D.E. delegates rebuked. They deemed that for the champion to be able to retain his title when both players have won 9 games apiece would give him the unfair advantage of requiring the challenger to win by 10-8. The rest is well-known; it was this impasse which led to Fischer's renunciation of his title and his continuing retirement from chess.

It was a legacy from this period that F.I.D.E. retained the minimum number of wins formula, draws not counting, but with the number set at 6 wins. We should point out that this by no means has lessened the

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number of draws, as the recent Karpov-Kasparov match amply illustrates. Even looking back into history, in the legendary Capablanca-Alekhine match of 1927, which was also played on the basis of 6 wins but had a 5-5 draw clause, there were 25 draws before Alekhine was able to wrest 6 wins.

All this experience should be borne in mind as we finally turn to our proposals. First of all, we agree with Fischer and his precedents that draws should not count, i.e. the total number of games should not be specified in advance. Drawing on the experience of the recent protracted match in Moscow, many have come out against discounting draws. However, one can argue that the lopsided score and the imperatives of consequent long-term strategies on both sides provoked the many draws, especially from the 27th game on. We shall see that according to our terms, the match would not have extended into such strained circumstances; indeed, Karpov would already have been declared the winner by the eighth game. Instead of being swayed by the arguments of the moment, we take the long-term, dispassionate view that, for example, winning one game in complications and managing to draw the rest is no real proof of superiority. However, we shall make some pragmatic proposals at the end of this article about delimiting the total number of games.

Let us pursue this idea of demonstrable superiority. It may well happen that the outcome of a match may not depend on the winner's superiority, but rather on a random bias of the score. Even if during the entire match, the rivals are equal in strength (i.e. their probabilities of winning each game coincide), various patterns of victories and defeats have equal and small but non-zero probability. Thus, the difference between the rivals' scores has random fluctuations.

At any given juncture in a match, let n be the total number of decisive games thus far, and k the difference between the opponents' cumulative scores at that point. Simple probabilistic calculations (see the appendix) show that, for opponents of equal strength, k^2 equals n on average. Hence, there is no reason to believe that the leading player is really stronger than his opponent until k^2 exceeds n . To discount initial fluctuations, we exclude $n = 1$ and $n = 2$. (Excluding $n = 3$ also may be considered more conclusive; we shall discuss this option at the end of this article.) Thus, with few exceptions, we propose that the match be terminated and the leading player declared winner when k^2 exceeds n . Counting only the decisive games, the first few such scores are: 3-0, 4-1, 5-2, 7-3, 8-4, 9-5, 11-6, etc. For example, 9-5 is a winning score since $9+5=14$ is n , and $9-5=4$ is k , and $4^2 = 16$ is greater than 14.

In proposing the 9-9 draw clause, Fischer and his precedents implicitly recognized the need for the challenger to demonstrate a tangible superiority if he is to take the title. We in fact require much greater differences than 10-8. Although the present F.I.D.E. rules require 6 wins, our proposal only requires 3 wins if the opponent has won none; 4 is necessary if the opponent has won 1; 5 is necessary if the opponent has won 2; and so forth. It is not the total number of wins (as before F.I.D.E. and also in the most recent matches), nor the difference in scores (as in the first F.I.D.E. matches), but the difference in scores in relation to the total number of decisive games which is germane.

Another game featuring prolonged matches between two players is tennis, and there the traditional scoring schemes do provide somewhat against random fluctuations: a game must be won by 2 points with a minimum of 4, a set must be won by 2 games (unless there is a tie breaker clause) with a minimum of 6, and only the match is won by simply the better score: 2 sets out of 3 (women) or 3 sets out of 5 (men).

Finally, chess players are well aware of a slight bias toward White, if only because he can to some extent determine the character of the game through his choice of opening. Thus, we propose that the total number of games (decisive or not) should be even, so that both players can have the same number of Whites.

To summarize our proposal:

Let k be the difference in the scores after n decisive games with n exceeding 2. A match should be terminated and the leading player declared winner whenever

- (a) k^2 exceeds n and
- (b) there has been an even number of games, decisive or not, played.

All this may sound rather abstract, and perhaps the empirical evidence from past world championship and candidates' matches ultimately provides the best argument for our proposal. Although we only provide a chart for world championship matches, it is a matter of record that the outcome of no single championship or candidates' match in chess history would have been reversed if our terms

had been invoked. Moreover, many matches would have been terminated much earlier than they actually were. About half of the candidates' matches would have been conclusive according to our terms, with the small total game limits working against demonstrating a tangible superiority. But fully two-thirds of the world championship matches would have been conclusive, and these would have been decided, on average, in less than half the actual number of games played, according to our terms. In fact, it is quite informative to go through the matches one-by-one and see just how early a clear superiority was demonstrated in each case. To be sure, some matches should have gone on, and this is quite consistent with our knowledge of the matches and rivals. For example, the second Steinitz-Chigorin match of 1892 had so many fluctuations that our terms would not have shortened it, and indeed, in the last game Chigorin could have evened the score, had he not, in a better position, overlooked a mate in two!

There are some nice nuances. Since we admitted the case $n=3$, a score of 3-0 for decisive games is sufficient. However, in the first Botvinnik-Smyslov match of 1954, Botvinnik had already achieved 3-0 by the fourth game – only for Smyslov to confound the situation by winning the next four decisive games! Ultimately, Botvinnik was able to draw the match and retain his title. The condition (b) leads us back from the brink in two cases: Both in the Steinitz-Zukertort match of 1886 and in the first Euwe-Alekhine match of 1935, the score of decisive games stood at one point at 1-4 against the eventual winner, a losing score by condition (a). But only an odd number of games had been played, and in both cases, the very next game was won by the eventual winner, marshaling the White pieces! To paraphrase Oscar Wilde, life imitates mathematics – rather grudgingly here, but nonetheless.

Let us conclude by discussing some options. The winning score of 3-0 for decisive games may seem too small and problematic to many players, but we have a logical reason for admitting it: Suppose that a match is permitted to continue after one of the players has achieved 3-0, and in the next four decisive games, he loses, then wins, then loses, and then wins. He has now achieved the winning score of 5-2, yet the even result of the last four games did not really demonstrate any further superiority beyond his 3-0 score. Hence, in admitting 5-2, we should admit 3-0.

For matches that need to be reasonably expedited, like the quarter-final candidates' matches at the beginning of the match cycle to decide the challenger, 3-0 should certainly be allowed. However, for important matches like the world championship itself, requiring n to exceed 3 might be a more conclusive test of superiority. In that case, 4 wins are required if the opponent has won at most 1 (in case of 4-0, condition (b) may be ignored); 5 wins are required if the opponent has won 2; and so forth. The anomalous (though insufficient for changing the fate of the crown) progress of the 1954 Botvinnik-Smyslov match would no longer be relevant; disallowing 3-0 would make the entire match inconclusive according to our terms. Our chart of world championship matches denotes the games at which the match would have been decided including and excluding 3-0. The only other match where the exclusion of 3-0 would make a significant difference is the Karpov-Korchnoi match of 1981; Karpov secured 3-0 by the fourth game, but Korchnoi won a game soon after, and only by the tenth game would Karpov have met both the conditions (a) and (b) with 4-1.

The next option concerns the length of matches. We are well aware that not counting draws may lead to inordinately long matches, which may not be viable for practical reasons. Thus, we can entertain the following sort of modification for world championship matches: An ultimate limit on the total number of games to be played can be set, say at 36 games. If by then no player has achieved a winning score according to our terms, then the leading player can be declared a "conditional" winner. Since he had not really demonstrated his superiority convincingly, his rival is now given the right to a return match, say within one year. We do not foresee that this sort of clause will often be invoked, since the historical record shows that a large number of decisive games tend to occur early in a match, when the players are forcing the pace, and a winning score by our terms can usually be expected. As mentioned earlier, this kind of clause would not have been relevant for the recent Karpov-Kasparov match, since according to our terms, Karpov would have been declared a winner by the eighth game (or by the ninth game, if we exclude the 3-0 possibility).

Perhaps there are other variants, especially for different levels of match competition. But our main condition (a) has an appealing simplicity while at the same time having a new underlying motivation. In writing this article, we are hoping that the wider chess community will at least start to consider such proposals as a viable alternative to the more traditional schemes. We welcome any comments.

APPENDIX

At any particular juncture in a match, let n be the number of decisive games thus far and k the difference in the players' scores. We shall show that if the players are considered of equal strength, then $E(k^2) = n$, where E denotes the expected value:

Fixing one player for the discussion, let X_i for $1 \leq i \leq n$ be the (Bernoulli) random variable whose possible values are 1 or -1 depending on whether he wins or loses the i -th decisive game. Note that $\sum X_i = \pm k$, with the \pm depending on whether our player is winning or losing. Thus,

$$E(k^2) = E[(\sum X_i)^2] = E(\sum X_i^2) + \sum_{i \neq j} E(X_i X_j)$$

since E distributes over sums. For the first sum on the right, $E(\sum X_i^2) = E(\sum 1) = n$. Also, the X_i 's are mutually independent variables, as the players are assumed to remain equal in strength throughout the match. Thus, for the second sum on the right,

$$\sum_{i \neq j} E(X_i X_j) = \sum_{i \neq j} [E(X_i)E(X_j)]$$

since E distributes over products of independent variables. However, $E(X_i) = 0$ for players of equal strength, and so this entire sum vanishes. Hence, $E(k^2) = n$.

Of course, k^2/n also fluctuates around its expected value of 1. But even for an infinite sequence of games, these fluctuations never exceed the order of $\log \log n$, by the Law of Iterated Logarithms. Our proposal ignores these fluctuations, not only to keep the rule simple, but also because $\log \log n$ grows very slowly. If n is the total number of electrons in the Universe, $\log \log n$ is still less than 6.

WORLD CHAMPIONSHIP MATCHES

In the following tabulation of official world championship matches, the winner is listed first, the scores given in the traditional way with draws counting 1/2, and the game-by-game results presented from the winner's point of view. Underlines indicate the game, if any, by which the match would have been decided earlier according to our terms. For example, in the 1978 Karpov-Korchnoi match, the score of 4-1 for decisive games was achieved by Karpov on the 17th game. This meets our condition (a), but only the 18th game is underlined, since to meet (b) one had to make sure that the 18th game was not won by Korchnoi. If a match would have been decided later had we excluded the possibility 3-0, a second underline indicates the game; only the 1954 Botvinnik-Smyslov match would become inconclusive with the exclusion of 3-0. Those matches which would have remained inconclusive altogether according to our terms are marked with *.

(Year Players Score (h = 1, 2))

1886 Steinitz-Zukertort 12.5-7.5	100001	1h1h11	0hh1h1	<u>11</u>				
1889 * Steinitz-Chigorin (I) 10.5-6.5	010110	011101	0111h					
1891 * Steinitz-Gunsberg 10.5-8.5	h1h001	1hh1h0	1hh0h1	h				
1892 * Steinitz-Chigorin (II) 12.5-10.5	0hh1h1	00h010	110101	01h11				
1894 Lasker-Steinitz (I) 12-7	1010hh	111 <u>1</u> h	00110h	1				
1897 Lasker-Steinitz (II) 12.5-4.5	111 <u>1</u> h1	hhh110	01h11					
1907 Lasker-Marshall 11.5-3.5	111 <u>h</u> hh	h <u>1</u> hhh1	111					
1908 Lasker-Tarrash 10.5-5.5	11011 <u>h</u>	1hh010	1hh1					
1909 Lasker-Janowski (I) 8-2	h11 <u>1</u> 10	1h11						
1910 * Lasker-Schlechter 5-5	hhhh0h	hhh1						
1910 Lasker-Janowski (II) 9.5-1.5	1hh11 <u>h</u>	<u>1</u> 1111						
1921 Capablanca-Lasker 9-5	hhhh1h	hhh11 <u>h</u>	<u>h1</u>					
1927 * Alekhine-Capablanca 18.5-15.5	1h0hhh	0hhh11	hhhhhh	hh1hhh	hhhh0h	h1h1		
1929 Alekhine-Bogolyubov (I) 15.5-9.5	1hh010	1h <u>1</u> h1	00h110	1h11hh	h			
1934 Alekhine-Bogolyubov (II) 15.5-10.5	h1h1hh	hh101 <u>h</u>	hhh11h	hh1h00	1h			
1935 * Euwe-Alekhine (I) 15.5-14.5	0100hh	0101h1	h1h0hh	011hhh	110hhh			
1937 Alekhine-Euwe (II) 15.5-9.5	01hh01	1h <u>1</u> hh	01hh0h	hh11h1	1			
1951 * Botvinnik-Bronstein 12-12	hhhh01	1hhh01	hhhh0h	1h001h				
1954 Botvinnik-Smyslov (I) 12-12	11h <u>1</u> hh	0h0001	1011hh	h0hh0h	(inconclusive without 3-0)			
1957 * Smyslov-Botvinnik (II) 12.5-9.5	1hh001	h1hhh1	0hhh1h	h1hh				
1958 Botvinnik-Smyslov (III) 12.5-10.5	111 <u>h</u> 0 <u>1</u>	hhhh01	h10hh1	0hh0h				
1960 Tal-Botvinnik (I) 12.5-8.5	1hhhh1	100h1h	hhhh1 <u>h</u>	1hh				
1961 Botvinnik-Tal (II) 13-8	101hhh	101 <u>1</u> 10	1h1h01	0h1				
1963 Petrosian-Botvinnik 12.5-9.5	0hhh1h	1hhhhh	h01hh1	<u>1</u> hhh				
1966 * Petrosian-Spassky (I) 12.5-11.5	hhhhhh	1hh1hh	0hhhhh	01h10h				
1969 * Spassky-Petrosian (II) 12.5-10.5	0hh11h	h1h00h	hhhh1h	101hh				
1972 Fischer-Spassky 12.5-8.5	001h11	h1h <u>1</u> 0h	1hhhhh	hh1				
1978 Karpov-Korchnoi (I) 16.5-15.5	hhhhhh	h1hh0h	11hh1 <u>h</u>	hh0hhh	hh100h	01		
1981 Karpov-Korchnoi (II) 11-7	11h <u>1</u> h0	hh1 <u>h</u> hh	01hhh1					
1984 Karpov-Kasparov 25-23	hh1hh1	<u>1</u> h1hhh	hhhhhh	hhhhhh	hh1hhh	h0hhhh	h...h00	