Problem Set 3

2019 Math Boot Camp for the Political and Social Sciences

Deeper Thinking

- 1. Let $\mathbf{u} = (a, b)$ and $\mathbf{v} = (c, d)$ be vectors. Join the ends of \mathbf{u} and \mathbf{v} to form a triangle. Show this triangle has area $\frac{1}{2}|ad bc|$.
- 2. Let ABC be a triangle. Let M be the midpoint of side AB and N be the midpoint of side AC. Show that the length of MN is half the length of BC.
- 3. Let a_1, a_2, \ldots, a_n be real numbers. Show $(a_1 + \ldots + a_n) \left(\frac{1}{a_1} + \ldots + \frac{1}{a_n}\right) \ge n^2$.

Some practice

- 1. Draw the vectors $\mathbf{u} = (1, 2)$, $\mathbf{v} = (4, -1)$. On the same set of axes draw:
 - (a) $\mathbf{u} + \mathbf{v}$
 - (b) 2**u**
 - (c) $-\frac{1}{2}$ **v**
 - (d) $\mathbf{v} \mathbf{u}$
- 2. For the vectors ${\bf u}$ and ${\bf v}$ above, compute:
 - (a) $||\mathbf{u}||$ and $||\mathbf{v}||$
 - (b) $||\mathbf{u} + \mathbf{v}||$ and verify the triangle inequality for these vectors.
 - (c) $|\mathbf{u} \cdot \mathbf{v}|$ and verify the Cauchy-Schwartz inequality for these vectors.

3. Let
$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$
 and $B = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}$ and compute:
(a) $A + B$
(b) det(A) and det(B)
(c) A^{-1} and B^{-1}

4. Solve the system below by (a) substitution, and (b) matrix inversion:

$$2x + 3y = 7$$
$$4x - 2y = 6$$

5. Read the exercises from Chapter 12 and 13 in [Moore-Siegel] and either do them or thoroughly convince yourself they're not worth your time.