## Problem Set 3

2019 Math Boot Camp for the Political and Social Sciences

## Deeper Thinking

1. Let $\mathbf{u}=(a, b)$ and $\mathbf{v}=(c, d)$ be vectors. Join the ends of $\mathbf{u}$ and $\mathbf{v}$ to form a triangle. Show this triangle has area $\frac{1}{2}|a d-b c|$.
2. Let $A B C$ be a triangle. Let $M$ be the midpoint of side $A B$ and $N$ be the midpoint of side $A C$. Show that the length of $M N$ is half the length of $B C$.
3. Let $a_{1}, a_{2}, \ldots, a_{n}$ be real numbers. Show $\left(a_{1}+\ldots+a_{n}\right)\left(\frac{1}{a_{1}}+\ldots+\frac{1}{a_{n}}\right) \geq n^{2}$.

## Some practice

1. Draw the vectors $\mathbf{u}=(1,2), \mathbf{v}=(4,-1)$. On the same set of axes draw:
(a) $\mathbf{u}+\mathbf{v}$
(b) $2 \mathbf{u}$
(c) $-\frac{1}{2} \mathbf{v}$
(d) $\mathbf{v}-\mathbf{u}$
2. For the vectors $\mathbf{u}$ and $\mathbf{v}$ above, compute:
(a) $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$
(b) $\|\mathbf{u}+\mathbf{v}\|$ and verify the triangle inequality for these vectors.
(c) $|\mathbf{u} \cdot \mathbf{v}|$ and verify the Cauchy-Schwartz inequality for these vectors.
3. Let $A=\left(\begin{array}{ll}1 & 2 \\ 0 & 1\end{array}\right)$ and $B=\left(\begin{array}{ll}2 & 3 \\ 2 & 1\end{array}\right)$ and compute:
(a) $A+B$
(b) $\operatorname{det}(A)$ and $\operatorname{det}(B)$
(c) $A^{-1}$ and $B^{-1}$
4. Solve the system below by (a) substitution, and (b) matrix inversion:

$$
\begin{aligned}
& 2 x+3 y=7 \\
& 4 x-2 y=6
\end{aligned}
$$

5. Read the exercises from Chapter 12 and 13 in [Moore-Siegel] and either do them or thoroughly convince yourself they're not worth your time.
