Problem Set 4

2019 Math Boot Camp for the Political and Social Sciences

Deeper Thinking

1. Consider the Markov chain corresponding to the matrix $A = \begin{pmatrix} 0.3 & 0.6 \\ 0.7 & 0.4 \end{pmatrix}$ with initial state $v_0 = \begin{pmatrix} 13 \\ 0 \end{pmatrix}$. Find a vector $v = \begin{pmatrix} x \\ y \end{pmatrix}$ such that $Av = v$ and $x + y = 13$. Compute $v_1, v_2, v_3, \ldots$. What do you notice?

2. Find all matrices A such that for *every* matrix B we have $AB = BA$.

Some practice

1. Compute AB and BA for $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}$.

2. Confirm $AA^{-1} = I$ and $A^{-1}A = I$ for $A = \begin{pmatrix} 5 & 3 \\ 4 & 3 \end{pmatrix}$. Can you prove it in general?

3. Compute Av for:
   (a) $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.
   (b) $A = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$ and $v = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$.
   (c) $A = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

4. For each of the following matrices A, graph the vectors $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and $v_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ on the same set of axes as Av_1, Av_2, and Av_3. What geometric transformation do these matrices correspond to?
   (a) $A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$
   (b) $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
   (c) $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

5. Read the exercises from Chapters 12 and 14 in [Moore-Siegel] and either do them or thoroughly convince yourself they're not worth your time.