Problem Set 5

2019 Math Boot Camp for the Political and Social Sciences

Deeper Thinking

1. For a function $f(x)$ find a condition on $f''(x)$, the second derivative (i.e. the derivative of the derivative), that determines whether a given a point such that $f'(x) = 0$ is a maximum or minimum.

2. Inductively (i.e. step-by-step) with polynomials construct a function $f(x)$ such that

$$f(0) = 1, \quad f'(0) = 1, \quad f''(0) = 1, \quad f'''(0) = 1, \quad f''''(0) = 1, \quad f'''''(0) = 1, \quad \ldots$$

and so on. Graph it. Do you notice anything?

3. Inductively (i.e. step-by-step) with polynomials construct a function $f(x)$ such that

$$f(0) = 0, \quad f'(0) = 1, \quad f''(0) = 0, \quad f'''(0) = -1, \quad f''''(0) = 0, \quad f'''''(0) = 1, \quad \ldots$$

and so on. Graph it. Do you notice anything?

Some practice

1. Find the derivative of the following functions from (i) the definition and (ii) linearity and the power rule:

   (a) $f(x) = 5$
   (b) $f(x) = x$
   (c) $f(x) = (x+1)^2 + 1$
   (d) $f(x) = 2x - x^2$
   (e) $f(x) = (x-3)(x+2) + x$

2. Graph the function $f(x) = x^2$ and its tangent lines at $x = 1$ and $x = -1$.

3. Graph the function $f(x) = x^3 - 3x^2 + 4$ on one set of axes and its derivative $f'(x)$ on a set directly below it. What are the x-intercepts of $f'(x)$? What do they correspond to on the graph of $f(x)$?

4. A farmer is building a rectangular pen for her chickens, using an existing brick wall as one of the sides. She has 40m in total of wire fencing to use. What is the maximum area of pen she can make?

5. Read the exercises from Chapter 5 in [Moore-Siegel] and either do them or thoroughly convince yourself they're not worth your time.