Problem Set 6

2019 Math Boot Camp for the Political and Social Sciences

Deeper Thinking

1. Some textbooks define $f(x) = e^x$ as the unique function such that it is its own derivative and $f(0) = 1$. They then define

$$\cos(x) = \frac{1}{2}(e^{ix} + e^{-ix}) \qquad\qquad \sin(x) = \frac{1}{2i}(e^{ix} - e^{-ix}),$$

   where $i^2 = -1$ is a square root of -1. Use these definitions to prove the derivative formulas for $\sin(x)$ and $\cos(x)$.

2. Compute $f(0), f'(0), f''(0), \ldots$ for $f(x) = \sin(x)$ and $f(x) = e^x$. Compare to the problems on the previous set. Any observations?

3. Prove the properties of the derivative we have introduced (i.e. linearity, the Chain Rule and the Product Rule) using the definition.

Some practice

1. Compute the following derivatives:

   (a) $f(x) = \dfrac{1}{x} + \sqrt{x}$

   (b) $f(x) = 2e^x - 5\log(x)$

   (c) $f(x) = \sin(x) + \cos(x)$

2. Compute the following derivatives using the Chain Rule or Product Rule as necessary:

   (a) $f(x) = e^{\sin(x)}$

   (b) $f(x) = (x^2 + 3\sqrt{x})\log(x)$

   (c) $f(x) = \log(\cos(x))$

3. The population of the city of Notsob is given by $P(t) = e^{8t - t^2}$, where t is the years since 2015.

   (a) When does the city reach its maximum population?

   (b) What is that population?

   (c) Does the population of the city ever reach zero?

4. Read the exercises from Chapters 6 and 8 in [Moore-Siegel] and either do them or thoroughly convince yourself they're not worth your time.