

## Problem Set 7

2019 Math Boot Camp for the Political and Social Sciences

### Deeper Thinking

1. Compute  $\sum_{k=0}^n \frac{1}{2^k}$ . Can you make sense of  $\sum_{k=1}^{\infty} \frac{1}{2^k}$ ?
2. Consider the function  $f(x) = e^x$ . Approximate the integral  $\int_0^5 e^x dx$  via rectangles 1 unit wide. Find a better approximation via trapeziums (i.e. the top can be an angled line) 1 unit wide. Is there any way to do better? Can you approximate by a quadratic?

### Some practice

1. Graph the function  $f(x) = 25 - x^2$ . Draw rectangles of width 1 underneath it between  $x = 0$  and  $x = 5$  and add them up to approximate the area under the graph. Can you get a better approximation with thinner rectangles?
2. Compute the following integrals:
  - (a)  $\int 3dx$
  - (b)  $\int x - 2x^2 dx$
  - (c)  $\int_0^5 (x+1)^2 dx$
3. Compute the areas under the following graphs between the given  $x$ -values:
  - (a)  $f(x) = 3$  between  $x = 1$  and  $x = 7$ .
  - (b)  $f(x) = 2x + 2$  between  $x = 0$  and  $x = 3$ .
  - (c)  $f(x) = 8x - x^2$  between  $x = 0$  and  $x = 8$ .
4. Compute the average value of  $f(x) = x^3 - 3x + 2$  between  $x = 2$  and  $x = 4$ .
5. The speed of a certain car is given by the function  $f(t) = 1 + 2t + 3t^2$  over the course of a journey between times  $t = 0$  and  $t = 4$ . What is the total distance travelled?
6. Read the exercises from Chapter 8 in [Moore-Siegel] and either do them or thoroughly convince yourself they're not worth your time.