Introduction to XPPAUT "Lab"

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Abstract

In this lab, we will explore the Lorenz model from the xppall/ode file that was downloaded with XPPAUT. We will make a bifurcation diagram for this model and use this to give us intuition about the dynamics of the full system in different parameter regimes. Then we will make and explore our own .ode file for the van der Pol oscillator.

1 The Lorenz model

To begin:

1. Start your X-server
2. Click and drag lorenz.ode to xpp.bat. This should open XPP.
3. On the left side of the open window is a menu- choose **Initialconds**, **(G)o** and you should see a very interesting orbit (the strange attractor ooohh!) **Remark:** You can avoid repeatedly pointing and clicking by hitting the keys indicated by capital letters, i.e. to do step 3 hit **i** then **g**.
4. Notice the blue buttons at the top of the window marked ICs, BCs, Delay, Param, etc. You can click these to open the current initial conditions, boundary conditions, any delay, parameters, etc. Of particular interest to us are the parameters. Try changing the parameter "r" and see what happens. In between runs, you may want to **Erase** the phase space (see menu).

1.1 Setting up XPP for AUTO

In order to effectively use AUTO starting from XPP we need the system to converge on a stable fixed point. Then AUTO will vary whatever parameter we choose and in what direction we choose to continue this fixed point in that parameter in find bifurcations.

5. We need to converge on a stable fixed point of the system. Set $r = 0.5$ in the Parameters window and hit "Ok".

6. Go to the **nUmerics** menu and hit **Total**. Increase the total integration time to 300 and hit Enter on your keyboard. We need to run the system long enough to be confident that it has converged. To leave this sub-menu, hit the Escape key on your keyboard.

7. Go to **Viewaxes**, **2d** and set your horizontal axis to **t** (time) and the vertical axis to **x**. Set xmin=0, ymin=-20, xmax=300, ymax=20. To leave this menu, you can click "Ok" or hit the Tab button on your keyboard. **Remark:** The Tab button is usually equivalent to clicking "Ok" in XPPAUT menus.

8. Now run the system (**Initialconds**, **Go**). It appears to have converged on a fixed point. To be sure hit (**Initialconds**, **Last**). This uses the last data point computed by XPP as the new initial condition.

9. Now hit the blue "Data" button at the top of the screen. You can page up and down to see if you XPP believes you are in fact on a fixed point. In this case, since the fixed point is at the origin the system has converged if you see all zeroes.

Remark 1. In theory, AUTO can start continuation from a periodic orbit if you converge to it in XPP first. However, I have never successfully done this.

Remark 2. In the nUmerics menu one can set Dt to be negative, which reverses time. This may help if you want to start AUTO on an unstable fixed point.

1.2 Using AUTO

Once you are sure that the system has converged to the fixed point, you are ready to run AUTO for the first time.

10. Choose **File**, **Auto** and the AUTO interface should pop up.

11. In the AUTO window, choose **Parameter**. We want r to be the bifurcation parameter, so make sure *Par1 is set to r. Hit Tab (or Ok).

12. Choose **Axes**, **hI-lo**. Hi-lo just means periodic orbits will be plotted as a pair of curves corresponding to the maximum and minimum value of the plotted variable along the orbit.

13. In the hI-lo menu, choose the Y-axis to correspond to the variable x. Set xmin=0, ymin=-20, xmax=30, ymax=20. Close the window.

14. Choose the **Numerics** menu. Set **Par max**=30. AUTO will stop running if the parameter of interest (r) exceeds "par max". Close the menu. For more information on the other items in the AUTO Numerics menu, see Bard Ermentrout's website (I sent the link on email).

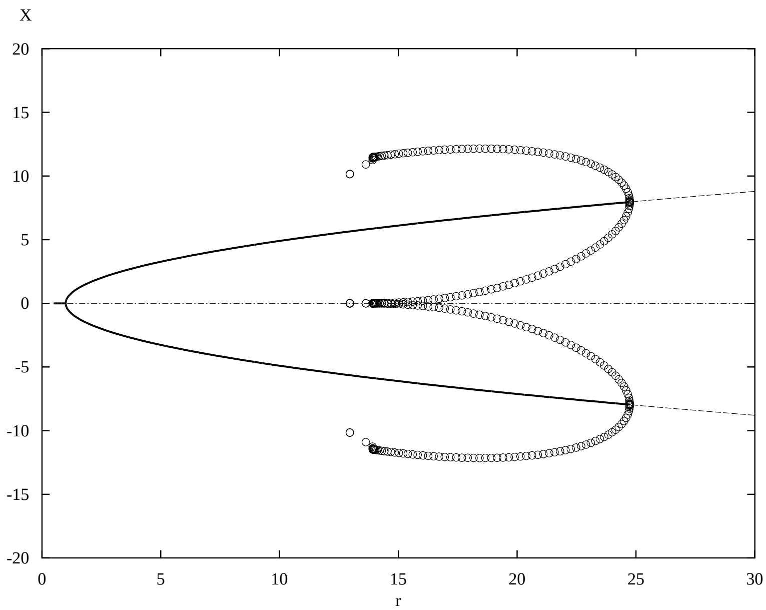
15. Now choose **Run**, **Steady state**. You should see a pitchfork bifurcation as r is increased. The outer, darker branch has stable fixed points and the center branch has unstable fixed points. It also appears that the stable branch loses stability near the right side of the diagram. Let's investigate.

16. Choose **Grab**. Hit the **Tab** button until you see the cross-like cursor over the location where one of the stable fixed points loses stability. Hit **Enter** to "choose" this point. On the bar at the bottom of the window under the letters **Ty** you should see the letters **HB**. This indicates that a Hopf Bifurcation occurs at this point

17. Now hit **Run**, **Periodic**. You should see two branches corresponding to unstable periodic orbits of the system.

18. Now do the same continuation at the *other* Hopf bifurcation, i.e. **Grab**, **Tab** until you get there, **Enter**,**Run**, **Periodic**.

Below is a picture of the bifurcation diagram I generated by following these steps.



You can save your own bifurcation diagrams using **File**, **Postscript**.

1.3 Optional: Using the information from AUTO to explore the phase space

From the bifurcation diagram, we might expect that for any choice of r between 1 and 14, the system decays to one of the two stable equilibrium points. For $r > 14$ the behavior will be more complicated because of the two families of unstable limit cycles, and we know that at some point the dynamics converge to a strange attractor.

20. **Close** the AUTO window so that only XPP is still open.

21. Set the XPP window to its original settings: **Viewaxes**, **3D**, xmin=-20, xmax=20, ymin=-30, ymax=30, zmin=0, zmax=50, xlo=-1.5, xhi=1.5, ylo=-2, yhi=2.

22. Try some different values of r and initial conditions. Can you figure out where the strange attractor dynamics "start"?

2 The van der Pol Oscillator

One version of the van der Pol oscillator is given by the following 2-dimensional system:

$$\dot{x} = y - \left(\frac{1}{3}x^3 - x\right)$$
$$\dot{y} = \varepsilon(a - x).$$

Here $0 < \varepsilon << 1$ and a are parameters. We will now create our own .ode file for this system.

1. Open the text file Lorenz.ode. Copy the text and paste it into a new text document. We will modify this for our new .ode file.

Here is the data that I entered into the text window:

```
# van der Pol oscillator
init x=2 y=4
par eps=0.1 a=1.3
x' = y - (1/3 * x^3 - x)
y' = eps * (a - x)
@ dt=.025, total=100, xplot=x,yplot=y, axes=2d
@ xlo=-3, xhi=3, ylo=-3, yhi=3
@ maxstor=20000
done
```

2. Once you have entered the text, save the file as “vanderpol.ode”.

3. Open vanderpol.ode with XPP.

4. You can view the nullclines of the system by hitting **Nullclines**, **New**.

4. Try varying the parameter a just above and below 1. What do you notice? We may expect there to be a Hopf bifurcation.

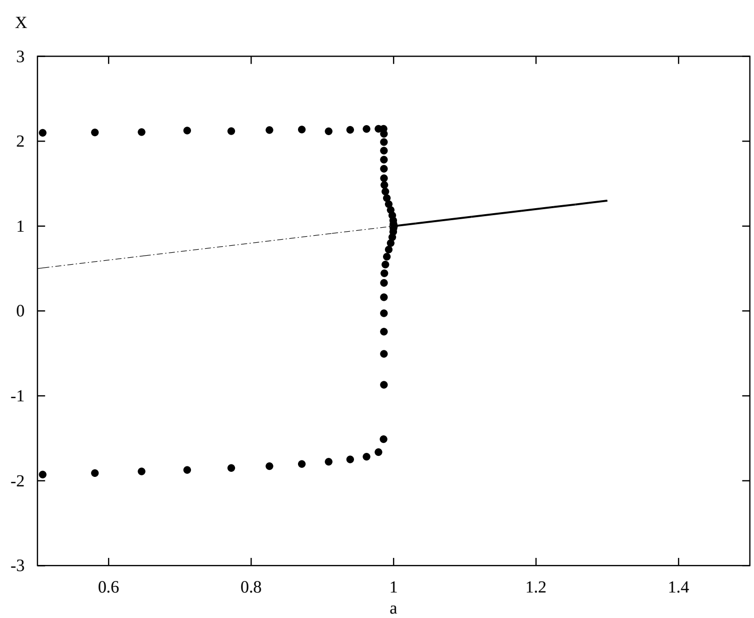
5. Choose a value of a for which the system has a stable fixed point (Hint: $a > 1$ will work) and make sure you have converged on this fixed point.

6. Now open AUTO (**File**, **AUTO**). Make sure the bifurcation **Parameter** is set to a and set up an appropriate viewing window using **Axes**, **hI-lo**. I used a horizontal axis between 0.5 and 1.5, and a vertical axis between -3 and 3.

7. Since we expect the Hopf bifurcation to occur to the left of our current value of a we need to tell AUTO to decrease the parameter. In the **Numerics** window, set **Dt**=-0.02.

8. **Run**, **Steady state**.

9. **Grab** the Hopf point and **Run**, **Periodic**. It's the canard explosion!!



Remark. You can make the curve look more connected by decreasing the size of the mesh (increase Ntst in Numerics menu) or by increasing the number of steps taken along any branch (increase Nmax). You can also save the AUTO data in a .dat file which can be read into MATLAB or Mathematica, and then "connect the dots" in one of those programs.