MA 122– Exam 1

July 18, 2011

In order to receive full credit, please show all work– You may use a calculator, but all derivatives and integrals must be computed by hand. Good luck!

Problem 1 Compute the indefinite integral: $\int x^2 e^{2x} dx$.

Solution Use substitution: Let $u = x^2$, $du = 2xdx$.

$$\begin{aligned} \int x^2 e^{2x} dx &= \int e^u du \\ &= e^u + C \\ &= e^{x^2} + C \end{aligned}$$

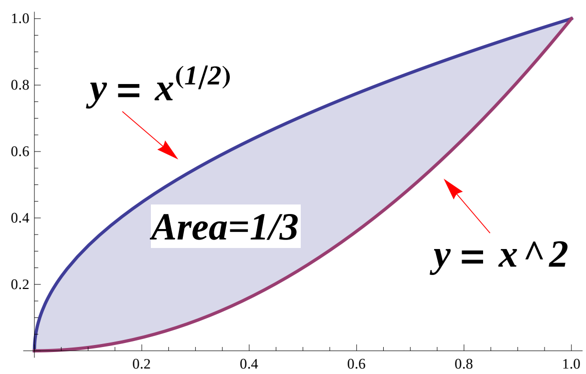
Problem 2 Let $f(x) = x^2$ and $g(x) = \sqrt{x}$.

(a) Calculate the area between f and g on the interval $[0, 1]$.

Solution

$$\begin{aligned} \text{Area} &= \int_0^1 (\sqrt{x} - x^2) dx \\ &= \left(\frac{2}{3} x^{3/2} - \frac{1}{3} x^3 \right) \Big|_0^1 \\ &= \frac{1}{3} \end{aligned}$$

(b) Sketch f and g on the same set of axes. Label your graph and indicate the quantity you found in part (a).



Problem 3 Let $f(x, y) = (x-1)^2 + y^2$.

(a) Find and classify all critical points of f. Justify your answer using the two "Theorems" from class.

Solution Critical points satisfy $\frac{\partial f}{\partial x} = 0$ or $\frac{\partial f}{\partial y} = 0$:

$$\begin{aligned} \frac{\partial f}{\partial x} &= 2(x-1) \\ &= 0 \\ \Rightarrow x &= 1 \end{aligned}$$

On the other hand,

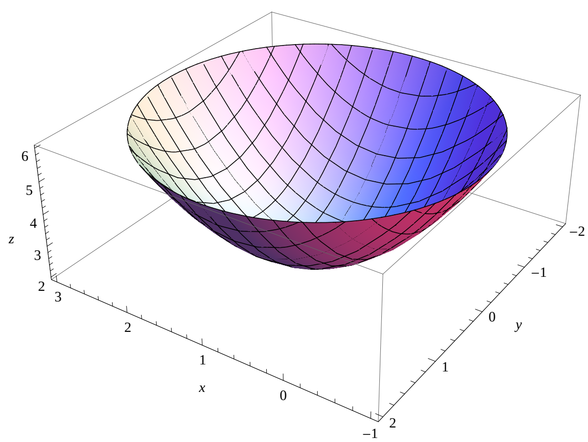
$$\begin{aligned} \frac{\partial f}{\partial y} &= 2y \\ &= 0 \\ \Rightarrow y &= 0 \end{aligned}$$

and so we see the only critical point of f is $(1, 0)$. To classify this point we calculate the second partial derivatives, and use Theorem 2 from p469:

$$f_{xy}(1,0) = 0,\ f_{xx}(1,0) = 2,\ f_{yy}(1,0) = 2$$

and so $AC - B^2 = 2(2) - 0^2 = 4 > 0$. Therefore the critical point is a *local minimum* (actually a global minimum in this case).

(b) Sketch a graph of f in the 3D coordinate system.



The graph of f is a paraboloid which opens upward and with vertex at $x = 1$, $y = 0$, $z = 0$.