

MA 122– Take-Home Quiz

August 4, 2011

The quiz is due **Monday, August 8** at 1pm. **No late quizzes** will be accepted. In order to receive full credit, please show all work and justify your answers. You may use the text, your notes, a calculator, and the internet but your work must be your own. Do not work together. Good luck!

Problem 1 The zombie apocalypse is coming! Research has shown that once the apocalypse hits, the human population of the greater Boston area will decrease at a rate proportional to both the population and the difference between the population and some limiting value M .

(a) Write a differential equation that models the population (in millions) of greater Boston at time t (in days). Choose a value of M that you think is reasonable, and explain why you chose this.

(b) Assume that the population of the greater Boston area is 4.5 million initially, and that 0.5 million people have died just 2 days into the apocalypse. Find the particular solution to your model from part (a).

(c) What is the human population one week after the start of the apocalypse?

(d) Sketch a graph of the solution you found in part (b) and discuss the long term behavior of this solution. Include important labels and indicate the solution to part(c) on the graph.

(e) Assuming no babies are born during the first week of the apocalypse, and all who perish become zombies immediately, what is the zombie population after one week?

(f) Will the Boston area human population ever go extinct? Explain why or why not. Is it possible to modify your model from part (a) to allow for this possibility?

(g) Another research facility claims that the model from part(a) is not entirely correct. They say that starting one week after the beginning of the apocalypse, a different population model must be used. They claim that the population decreases at a rate proportional to itself only.

Based on statistical predictions, they also suspect that one month after the start of the apocalypse the population will be half of what it was at onset. Assuming this to be true, (i) write a differential equation for the new population model (for $t \geq 7$) and (ii) find the particular solution. (iii) Sketch a graph of the solution.

(h) Based on the new model, when will the human population dip below 0.5 million? Indicate this result on your graph in part (g).