

Quiz 1

MA 225 A1, 5/23/12

1. Compute the line tangent to $f(x) = \sin(\pi x) + \frac{3x}{x^2+2}$ at $x = 1$.

Solution. To compute the tangent line we need a point on the line and the slope of the line. Since $f(1) = 1$, a point on the line is $(1, 1)$. To find the slope, we must calculate the derivative at $x = 1$:

$$\begin{aligned} f'(x) &= \pi \cos(\pi x) + \frac{(x^2 + 2)(3) - (3x)(2x)}{(x^2 + 2)^2} \\ \Rightarrow f'(1) &= -\pi + \frac{9 - 6}{9} \\ &= -\pi + \frac{1}{3}. \end{aligned}$$

The equation of a line in slope-intercept form is $y = mx + b$, and so in order to complete the problem we must identify the constant b :

$$\begin{aligned} 1 &= \left(-\pi + \frac{1}{3}\right)(1) + b \\ \Rightarrow b &= \pi + \frac{2}{3}. \end{aligned}$$

Thus, the equation of the tangent line is

$$y = \left(-\pi + \frac{1}{3}\right)x + \pi + \frac{2}{3}.$$