Quiz 5, MA 225 A1, 6/19/12

1. Consider $\int_0^3 \int_0^{\sqrt{9-x^2}} f(x,y)dydx$.
(a) Sketch the region of integration, and then rewrite the integral by switching the order of integration.

Solution. The region of integration is the quarter of the disk of radius 3 centered at the origin in the first quadrant. By changing the order of integration we arrive at

$$\int_0^3 \int_0^{\sqrt{9-x^2}} f(x,y)dydx = \int_0^3 \int_0^{\sqrt{9-y^2}} f(x,y)dxdy.$$

(b) Rewrite the integral in terms of polar coordinates (r, θ).

Solution. $\int_0^{\frac{\pi}{2}} \int_0^3 f(r\cos\theta, r\sin\theta)rdrd\theta$

2. Set up and evaluate the triple integral of $f(x,y,z) = 2 - z$ over the region $D = \{(x,y,z) : 0 \le x \le 2,\ 0 \le y \le 3,\ 0 \le z \le 1\}$. Sketch the region of integration D.

$$\begin{aligned}\int\int\int_D f dV &= \int_0^1 \int_0^3 \int_0^2 (2-z)dxdydz \\ &= \int_0^1 \int_0^3 (4-2z)dydz \\ &= \int_0^1 (12-6z)dz \\ &= 12 - 3 = 9.\end{aligned}$$

The region of integration is a rectangular solid in the first octant of $\mathbb{R}^3$.