Quiz 6, MA 225 A1, 6/20/12

Consider the sphere S of radius p centered at the origin. Use a triple integral to represent the volume of S in (a) spherical coordinates, (b) cylindrical coordinates and (c) Cartesian coordinates. Then evaluate one of these three integrals to calculate the volume.

Solution. (a) Volume $= \int_0^{2\pi} \int_0^{\pi} \int_0^p \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

(b) Volume $= \int_0^{2\pi} \int_0^p \int_{-\sqrt{p^2-r^2}}^{\sqrt{p^2-r^2}} dz \, r \, dr \, d\theta$

(c) Volume $= \int_{-p}^p \int_{-\sqrt{p^2-x^2}}^{\sqrt{p^2-x^2}} \int_{-\sqrt{p^2-x^2-y^2}}^{\sqrt{p^2-x^2-y^2}} dz \, dy \, dx$

The triple integral in part (a) is probably the easiest to compute. The result is $\frac{4}{3}\pi p^3$.