LAB 1

Lab due Thursday, July 15th, 2010 in class. Late labs will not be graded.

You may use any technology that you have available: a spreadsheet, Mathematica, Matlab, programmable calculator, etc. You will be graded on exactly what is asked for in the instructions below. You need not turn in any additional data, graphs, paragraphs, etc. You should submit only what is called for, and in the order the questions are asked.

IMPORTANT: The work you submit should be your own and nobody else's. Any exceptions to this will be dealt with harshly.

Introduction: In this lab, you will need to use two numbers, A and B. These numbers are derived from your BU-student-ID (if you don't have a BU-ID come see me) as follows. The number A is the last nonzero number in your student ID, while the number B is the second last nonzero number in your ID. For example, if your student ID is 123-45-6789, then $A = 9$ and $B = 8$. But if your ID is 100-20-3000, then $A = 3$ and $B = 2$

We have seen in class how to use Euler's Method to approximate the solutions of differential equations. We have also seen that Euler's method usually increases in accuracy if Δt is chosen smaller. In this lab you will investigate <u>how</u> the accuracy of Euler's method changes as the step size becomes smaller.

Answer each of the following questions in order:

0. Give your name and student ID. Specify explicitly A and B. 1. Consider the initial value problem

$$\frac{dy}{dt} = -4t + 1, \quad y(0) = A$$

where the constant A is determined from your student ID as above. Find the exact solution $y(t)$ to this initial value problem and determine the value $y(1)$. Be sure to check that your answer here is correct and show this computation explicitly. If your answer here is wrong, the rest of this lab makes no sense and grading will stop at this point.

2. Use Euler's method with a step size of $\Delta t = 0.1$ to approximate $y(1)$. That is, using Euler's Method, compute in succession $(t_0, y_0), (t_1, y_1), ..., (t_{10}, y_{10})$ where $t_0 = 0, y_0 = A$ and $t_{10} = 1$ so that y_{10} is an approximation of $y(1)$. List in table form the values you find for $(t_0, y_0), (t_1, y_1), ..., (t_{10}, y_{10})$. Highlight your approximate value for $y(1)$ using this step size. What is the error here (the difference between your approximate value and the actual value of $y(1)$)?

3. Repeat question 2 with a step size of $\Delta t = 0.05$, i.e., with twice as many steps.

4. Repeat question 2 with a step size of $\Delta t = 0.01$, i.e., with ten times as many steps as in question 2. You need not present all of the data here; just give the approximation to $y(1)$ that you find using this step size and the error.

5. In a brief essay (no more than one page), discuss the improvement of the accuracy of Euler's Method as you make the step size smaller by a factor of $\frac{1}{2}$ and $\frac{1}{10}$. How does this affect your approximation of $y(1)$? By how much does your approximation improve percentage-wise?

6. Now consider a second initial value problem

$$\frac{dy}{dt} = 3y + 4, \quad y(0) = \frac{B}{100}$$

where the constant B is determined from your student ID again as above. Remember to use $\frac{B}{100}$, not just B. Now repeat questions 2-5 for this initial value problem. Remember to check first that your exact solution of this initial value problem is absolutely correct (using this value). Your results to the second part will not be as "clean" as in the first part. In your essay, discuss how close these two different results are.