LAB 3

Lab due Wednesday, August 4th, 2010 in class. Late labs will not be graded.

You may use any technology that you have available: a spreadsheet, Mathematica, Matlab, programmable calculator, etc. You will be graded on exactly what is asked for in the instructions below. You need not turn in any additional data, graphs, paragraphs, etc. You should submit only what is called for, and in the order the questions are asked.

IMPORTANT: The work you submit should be your own and nobody else's. Any exceptions to this will be dealt with harshly.

Instructions.

In this lab you will investigate a three parameter family of differential equations. Your goal is to provide an understandable picture of the "parameter space" of a three parameter family of linear systems (an analogue of the trace-determinant plane).

Here is the system:

$$\frac{dx}{dt} = ay$$

$$\frac{dy}{dt} = bx + cy$$

This system depends upon three parameters a, b, and c. Your goal is to provide an an accurate and comprehensible "picture" of the a, b,c-space, indicating the regions where this system has the various types of behavior (spiral sinks, repeated eigenvalues, zero eigenvalues, saddles, etc.). You can think of your task as dividing the parameter space into different regions where the system shows a particular behavior. Clear presentation of your solution is important in this lab. Please use different colors or shadings for the different regions. Keep in mind that you can use any technology that you see fit and the does not necessarily need to handed in on a two dimensional paper, be creative.

Answer the following questions about this system in order.

1. First consider the case $c = 0$. Compute the eigenvalues for this special case and determine the exact a,b-values where this system has different types of behavior, i.e., spiral sinks, sources, saddles, etc. Then draw an accurate picture of the a,b-plane, indicating the regions (i.e., the points (a,b)) where the two parameter family

$$\frac{dx}{dt} = ay$$
$$\frac{dy}{dt} = bx$$

has spiral sinks, sources, saddles, etc. Display all of the different types in your picture. Use different colors or shadings for different regions. Also indicate where you find the special situations: repeated and/or zero eigenvalues, etc.

2. Now repeat question 1 for some positive c value, say $c = 2$.

3. Describe in words and in pictures what happens to the picture in question 2 above when you take smaller c-values, with c between 0 and 2. Then describe what happens to the picture when you choose c-values larger than 2.Words and pictures suffice, but be creative.

4. Now repeat question 1 for some negative c-value, say $c = -2$.

5. Then repeat question 3 for $c = 2$-values in the interval $-2 < c < 0$ and $c < -2$.

6. Now try to draw a three dimensional version of this picture, with the c axis vertical and the a, b-plane perpendicular to this axis. Highlight the special cases where your system changes its type. This is tough to visualize. Maybe you can build a three-dimensional model of this space.

   Important Remark:

   You do not need to use technology for this lab, though you are free to do so if you wish. Have fun, let your mind go free, but nothing too wild. Do not harm any rabbits or foxes, and no expression of mathematical concepts through dance.