

## Final Exam: Sample Questions

1. Find the general solution of:

$$\frac{dy}{dt} = (y^2 + 1)t^2$$

2. Find the general solution of:

$$y'' + 3y' + 3y = 0$$

3. Find the general solution of:

$$\frac{dy}{dt} = y^2 - 4$$

4. Plot  $x(t)$  and  $y(t)$  for the solution curve shown on the figure below in blue

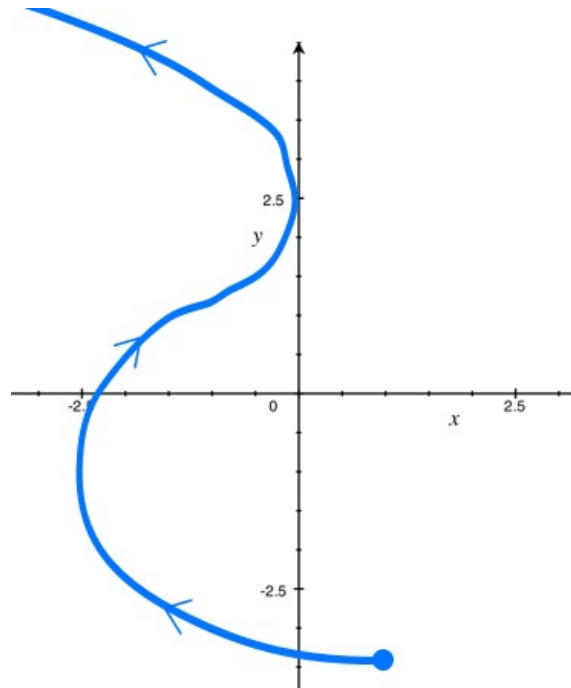


Figure 1: Solution curve

5. Find the general solution of

$$x' = 2x + y$$

$$y' = x + y$$

Also sketch the phase plane for this system. Draw the  $x(t)$  and  $y(t)$  graphs for the solution of this system starting at  $(1, 1)$ .

6. In a short essay (a page, two or three), describe as much as possible what bifurcation means. You should give at least one example of a bifurcation. Provide graphs as necessary.
7. Find all equilibrium points for the following system and determine their type. Then sketch the phase plane for this system, indicating the locations of all nullclines.

$$x' = y$$

$$y' = -y - \sin(x)$$

8. Write a brief essay about Euler's method. Explain the formula used in this method. Show a picture of what Euler's method is actually accomplishing.
9. Find the solution of the forced mass-spring system described by the second order differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 2\cos(-3t)$$

satisfying  $y(0) = y'(0) = 0$  and sketch the graph of  $y(t)$ . Determine the type of damping and in a sentence or two describe the fate of the oscillator as time passes.

10. One of the nonlinear systems of differential equations governing a laser is given by

$$\frac{dx}{dt} = 1 - x - 3xy$$

$$\frac{dy}{dt} = -y + 3xy$$

Using any and all means possible, describe as much of the phase plane for this system as you can.

11. [20 points] In this problem, restrict attention to the first quadrant ( $x, y \geq 0$ ).

For the following system of equations,

$$\frac{dx}{dt} = x(-x - 3y + 150)$$

$$\frac{dy}{dt} = y(-2x - y + 100).$$

- (a) sketch the nullclines,
- (b) sketch the phase portrait,
- (c) write a brief paragraph describing the behavior of solutions  
(show all the work you do to reach your answers).

12. **All or Nothing Questions.** Answers only. No partial credit.

- Find one particular solution to:

$$\frac{dy}{dt} = y^5 - 1.$$

- Find all equilibrium solutions of the differential equation:

$$\frac{dy}{dt} = y^3(t^2 - 4)(t^4 - 9t^2).$$

- Find all equilibrium solutions of the differential equation:

$$\frac{dy}{dt} = \frac{(y^2 - 4)}{y - 3}.$$