MA 226 Summer II 2010: Midterm 1

1. [15 points] First sketch the slope field for the differential equation:

$$\frac{dy}{dt} = \cos^2(y).$$

On the same picture, draw the graph of the solutions that satisfy $y(0) = 0$, $y(0) = \pi$ and $y(0) = 2\pi$.

2. [15 points] Find all equilibrium points for the differential equation:

$$\frac{dy}{dt} = y^4 - 5y^3 + 6y^2$$

and determine which are sinks, sources or nodes.

3. [25 points] Consider the family of differential equations, that depend on the parameter $a \in \mathbb{R}$:

$$\frac{dy}{dt} = y^2(y - a).$$

Find for what a value(s) a bifurcation occurs and sketch the bifurcation diagram.

4. [15 points] First find the general solution of the differential equation:

$$\frac{dy}{dt} + 3y = e^{2t}.$$

Then find the particular solution that satisfies $y(0) = 1$.

5. [20 points] First state the Existence and Uniqueness Theorem.

   Then determine which of the following differential equations satisfy the conditions of the Theorem. In the case(s) the conditions are not satisfied explain why:

   - $\frac{dy}{dt} + 3y = e^{2t}$
   - $\frac{dy}{dt} = \frac{y}{t^3}$
   - $\frac{dy}{dt} = \sin^2(y)$
   - $\frac{dy}{dt} = y^{1/3}$

6. [10 points] **All or Nothing Questions**. Answers only. No partial credit.

   - Find one particular solution to:

   $$\frac{dy}{dt} = y^{11} - 1.$$

   - Find all equilibrium solutions of the differential equation:

   $$\frac{dy}{dt} = y^2(t^2 - 4)(t^4 - 9t^2).$$

   - Find all equilibrium solutions of the differential equation:

   $$\frac{dy}{dt} = \frac{y+3}{y-1}.$$