MA 226 Summer II 2010: Midterm 2: July 22nd

1. [20 points] Solve the system of differential equations

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

   Sketch solutions in the phase plane that give a good picture of general solution behavior. Find the solution which at time zero is at $(1, 1)$ and sketch x and y for that particular solution as a function of time.

2. [30 points] Find the general solution for

$$\vec{\mathbf{Y}}' = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \vec{\mathbf{Y}}$$

   Sketch phase plane portrait. Solve the initial value problem $\vec{\mathbf{Y}}(0) = (1, 1)$ and sketch x and y for that particular solution as a function of time.

3. [20 points] Consider the family of differential equations, that depend on the parameter $a \in \mathbb{R}$:

$$\vec{\mathbf{Y}}' = \begin{pmatrix} 3 & 0 \\ 2 & a \end{pmatrix} \vec{\mathbf{Y}}$$

   Find for what a value(s) a bifurcation occurs and describe solution behavior before and after the bifurcation(s). Note that you do not need to describe solution behavior at the bifurcation value(s).

4. [20 points] Consider a predator pray model of rabbits and wolves living on a island.

$$\frac{dR}{dt} = kR - \alpha RF$$
$$\frac{dF}{dt} = -lF + \beta RF$$

Modify the model to reflect the following changes:

a) The wolves learn how to dress up as carrots and get twice as good at hunting the clueless rabbits.

b) The rabbits beef up and start throwing the wolves off the island.

c) The UN sends Baldur on a boat to the island. He gets the rabbits to stop mentioning that whole little red riding hood incident to the wolves and the wolves to start eating a new food source (tofa-bbit). Thus brokering a peace deal.

5. [20 points] Each of the figures on the following page go with one of the equations labeled (a)-(h). Match each figure with a pair of differential equations.

(a) $\frac{dx}{dt} = x$, $\frac{dy}{dt} = y^2 - 1$ (b) $\frac{dx}{dt} = x$, $\frac{dy}{dt} = 2y$ (c) $\frac{dx}{dt} = 2x$, $\frac{dy}{dt} = y$ (d) $\frac{dx}{dt} = -x$, $\frac{dy}{dt} = y^2 - 1$

(e) $\frac{dx}{dt} = 1 + y$, $\frac{dy}{dt} = 1 - x$ (f) $\frac{dx}{dt} = 2y - x$, $\frac{dy}{dt} = -y$ (g) $\frac{dx}{dt} = 2y + x$, $\frac{dy}{dt} = -y$ (h) $\frac{dx}{dt} = 1 - y$, $\frac{dy}{dt} = 1 + x$

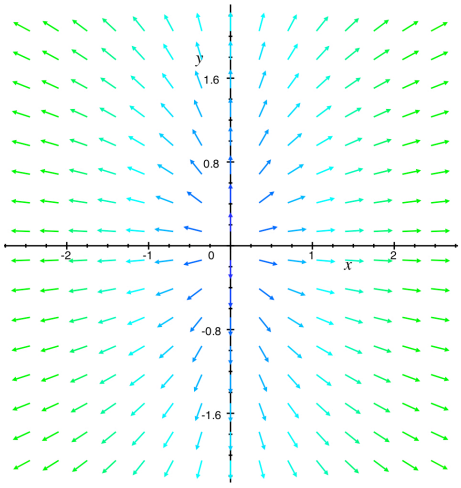

Figure 1:

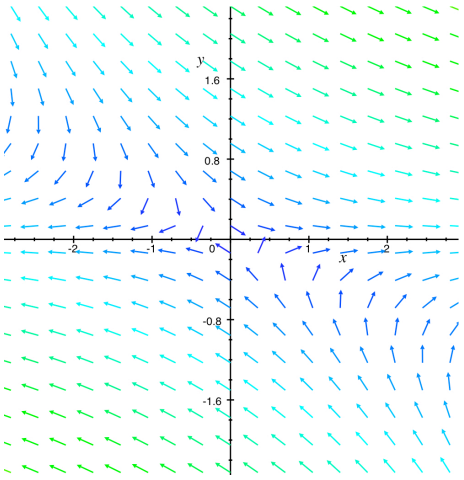

Figure 2:

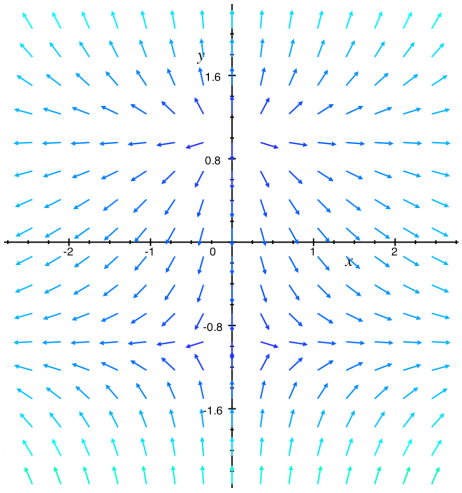

Figure 3:

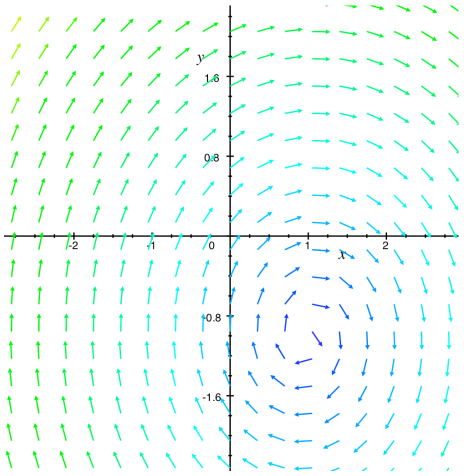

Figure 4: