# MA 226 Summer II 2010: Midterm 3: August 2nd

1. [10 points] Consider the system of differential equations

$$\frac{dx}{dt} = 5y - 3x$$
$$\frac{dy}{dt} = -2x - 2y.$$

Find all equilibrium points and determine the type of each equilibrium point(s).

2. [20 points] Consider the system of differential equations

$$\frac{dx}{dt} = y - x$$
$$\frac{dy}{dt} = x^3 - y.$$

Find all equilibrium points and determine the type of each equilibrium point(s).

3. [20 points] Find the general solution and sketch the phase plane portrait for

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y. \end{pmatrix}.$$

4. [20 points] Find the solution of the forced mass-spring system described by the second order differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 2e^{-3t}$$

satisfying $y(0) = y'(0) = 0$ and sketch the graph of $y(t)$. Determine the type of damping and in a sentence or two describe the fate of the oscillator as time passes.

5. [14 points] Consider the systems of equations, that depend on the parameter $a \in \mathbb{R}$:

$$\vec{\mathbf{Y}}' = \begin{pmatrix} 0 & 1 \\ -4 & -a \end{pmatrix} \vec{\mathbf{Y}}.$$

Sketch the corresponding path in the trace-determinant plane and determine at what $a$ values bifurcation(s) occur. Indicate with a figure on your sketch the different types of phase planes for this system.

6. [16 points] Below are nine second order differential equations and four graphs showing $y(t)$. Match the number of a graph with the differential equation for which it is a solution. In a sentence or two, tell me how you know that a particular equation goes with the corresponding graph. Each graph matches with exactly one equation.

| | | |
|---|---|---|
| (a) $y'' + y = \sin(t)$ | (d) $y'' + y = \sin(\frac{t}{4})$ | (g) $y'' + y = \sin(\frac{1}{10}t)$ |
| (b) $y'' + y = 2$ | (e) $y'' + y' + y = \sin(t)$ | (h) $y'' + y = \sin(\frac{3}{4}t)$ |
| (c) $y'' + y = -2$ | (f) $y'' + y' + y = \cos(t)$ | (i) $y'' + y = \sin(\frac{9}{10}t)$ |

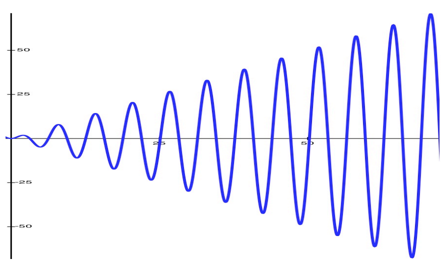

Figure 1:

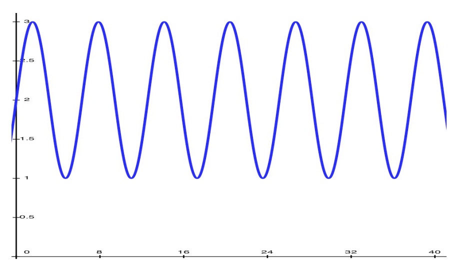

Figure 3:

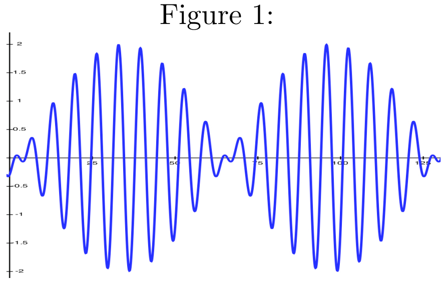

Figure 2:

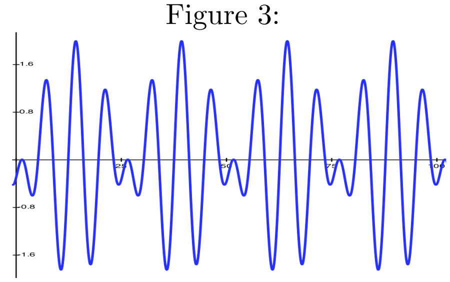

Figure 4: