

MA 226

Differential Equations

Baldur Hedinsson

baldur@bu.edu

office: MCS 168

Office hours: Tu. 13-14

Th. 13-14

MA 226 Maestro

Robert Devaney

- <http://math.bu.edu/people/bob/MA226/>
- google Devaney MA 226
- homework
- sample exams

Homework

- assigned daily
- not picked up or graded
- often reappears on exams

Today

- Section 1.1 page 14. #3,5,15,17
- last 30 min of class for homework

Course

- Three midterms
- Two highest scores count • 40%
- No make up

- Final exam • 40%
- Labs (tentative) • 16%
- Attendance • 4%
- grades posted on blackboard 100%

Course

- Differential Equations
 - Active research topic
 - Cannot solve many (most) diff eqs
 - Computers can help
 - Numerical error
 - Chaos

Modeling

- Differential Equations
 - Assumptions
 - Specify variables and parameters
 - Write out equations
- Solve
 - Analytical (old)
 - Qualitatively
 - Numerically

Example

Modeling Population Growth

- Assume: Rate of growth of population is proportional to population at present
- Predict: Population at any later time
- Variables:
 - $t = \text{time}$
= independent variable
 - $P = P(t)$
= Population at time t
= dependent variable
 - Predict $P(t)$

Unlimited Growth

- Rate of change is prop. to population

$$\left\{ \begin{array}{l} \frac{dP}{dt} = k \cdot P(t) \\ P(0) = P_0 \end{array} \right. \leftarrow \begin{array}{l} \text{diff. eq.} \\ \text{initial condition} \end{array}$$

- Solution technique: separate and integrate

Let's do it

Conclusion

$$P(t) = P_0 e^{kt}$$

$$P(t) \rightarrow \infty \text{ as } t \rightarrow \infty$$

$$P(t) = 0 \text{ if } P_0 = 0$$

constant solution, equilibrium solution

we can predict population at any time

Example

Given $P(0) = 100$ and $P(1) = 150$

$$P(10) = ?$$

again, we can predict any future population


again $P(t) \rightarrow \infty$ as $t \rightarrow \infty$

$$P(t) = 0 \text{ if } P_0 = 0$$

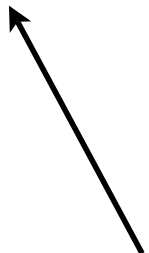
Qualitative Approach

$$\frac{dP}{dt} = 1 \cdot P$$

slope of
graph
of $P(t)$



we know
slope at
all times



Slope Field

1. Plot of slope of $P(t)$
at point (t, P)

2. Graph of $P(t)$
is tangent to
slope field

Summary

- Construct model
 - Assumptions
 - Variables and parameters
 - Write out equations
- Solve
 - Analytical (old)
 - Qualitatively
 - Numerically
- Specify behavior of all solutions