MA 226
Differential Equations

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MA 226 Maestro Robert Devaney

- http://math.bu.edu/people/bob/MA226/
  - google Devaney MA 226
  - homework
  - sample exams

Homework

- assigned daily
- not picked up or graded
- often reappears on exams

Today

- Section 1.1 page 14. #3,5,15,17
- last 30 min of class for homework

Course

- Three midterms
- Two highest scores count
- No make up

- 40%

- Final exam
- Labs (tentative)
- Attendance

- 40%
- 16%
- 4%

100%

- grades posted on blackboard

Course

- Differential Equations
  - Active research topic
  - Cannot solve many (most) diff eqs
  - Computers can help
  - Numerical error
  - Chaos

Modeling

- Differential Equations
  - Assumptions
  - Specify variables and parameters
  - Write out equations
- Solve
  - Analytical (old)
  - Qualitatively
  - Numerically

Example

Modeling Population Growth

- Assume: Rate of growth of population is proportional to population at present
- Predict: Population at any later time
- Variables:
  - t = time
    = indipendent variable
  - P = P(t)
    = Population at time t
    = dependent variable
  - Predict P(t)

Unlimited Growth

- Rate of change is prop. to population

$$\begin{cases} \dfrac{dP}{dt} = k \cdot P(t) & \longleftarrow \text{diff. eq.} \\ P(0) = P_0 & \longleftarrow \text{initial condition} \end{cases}$$

- Solution technique: separate and integrate

Let’s do it

Conclusion

$$P(t) = P_0 e^{kt}$$

$P(t) \to \infty$ as $t \to \infty$

$P(t) = 0$ if $P_0 = 0$

constant solution, equilibrium solution

we can predict population at any time

Example

Given $P(0) = 100$ and $P(1) = 150$

$$P(10) = ?$$

again, we can predict any future population

again $P(t) \longrightarrow \infty$ as $t \longrightarrow \infty$

$$P(t) = 0 \text{ if } P_0 = 0$$

Qualitative Approach

$$\frac{dP}{dt} = 1 \cdot P$$

slope of
graph
of $P(t)$

we know
slope at
all times

Slope Field

1. Plot of slope of $P(t)$ at point (t, P)

2. Graph of $P(t)$ is tangent to slope field

Summary

- Construct model
  - Assumptions
  - Variables and parameters
  - Write out equations
- Solve
  - Analytical (old)
  - Qualitatively
  - Numerically
- Specify behavior of all solutions