Day 11: July 14th

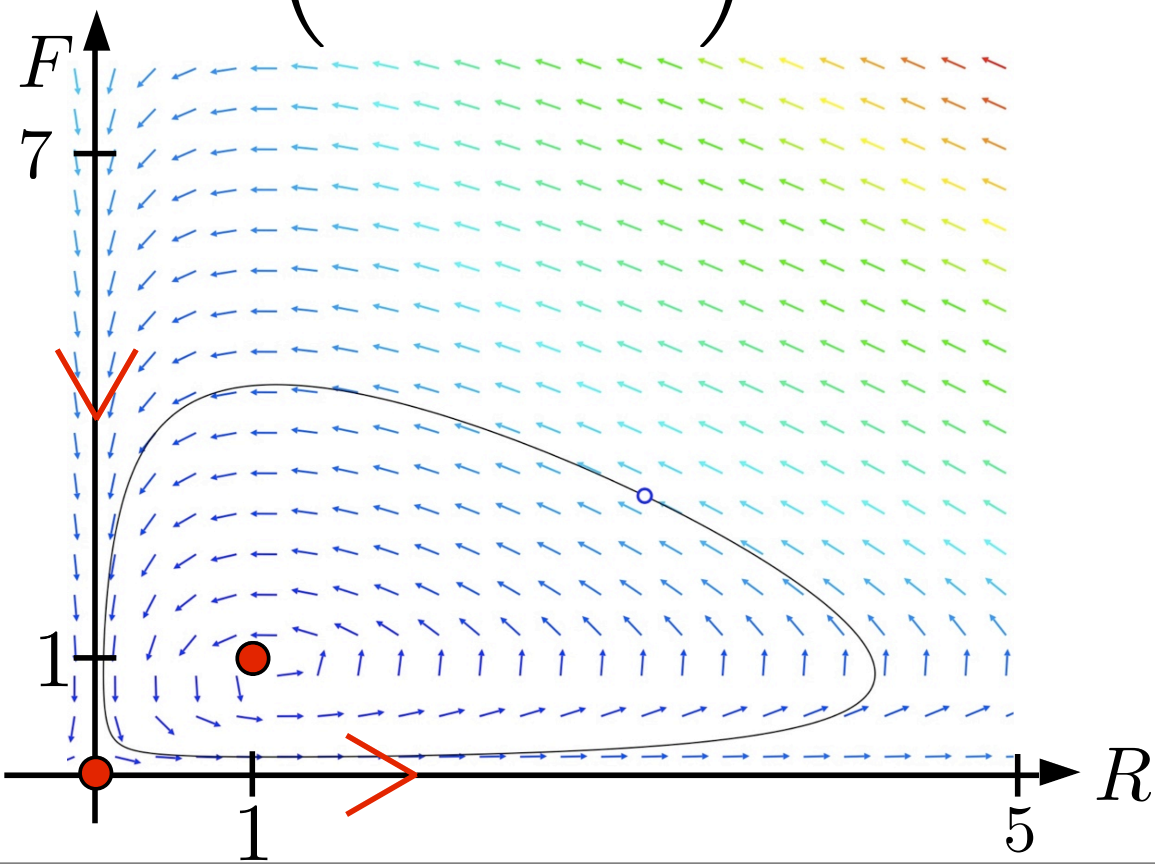
- **Chapter: 3.1 Properties of Linear Systems.**
  - Homework:
    - Page 252 #5, 9, 17, 25, 27, 29.

- **Chapter: 3.2 Straight Line Solutions.**
  - Homework:
    - Page 271 #1-7 odd, 11, 13, 21, 23.

- rabbit vs foxes: example 1

$$\frac{dR}{dt} = R \cdot \Big(1 - F\Big)$$

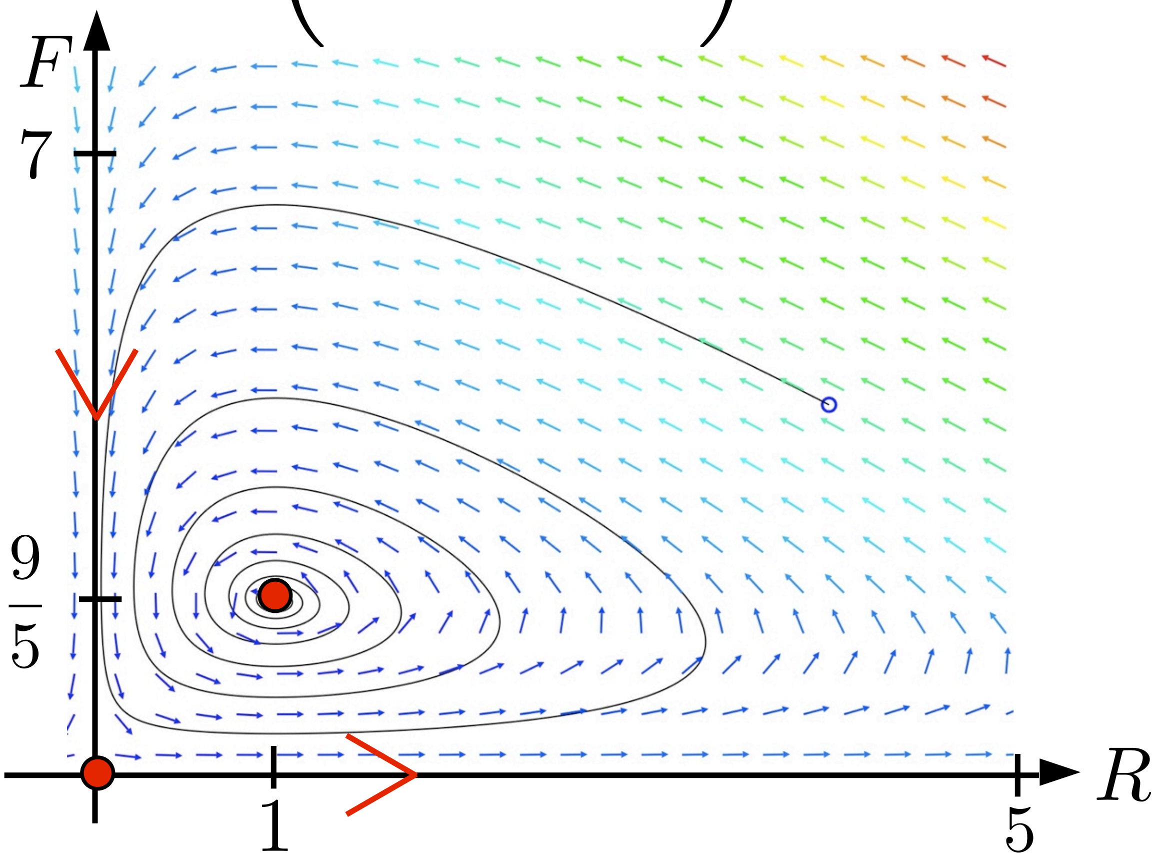
$$\frac{dF}{dt} = F \cdot \Big(-1 + R\Big)$$



- rabbit vs foxes: example 2

$$\frac{dR}{dt} = 2R \cdot \left(1 - \frac{R}{10}\right) - R \cdot F$$

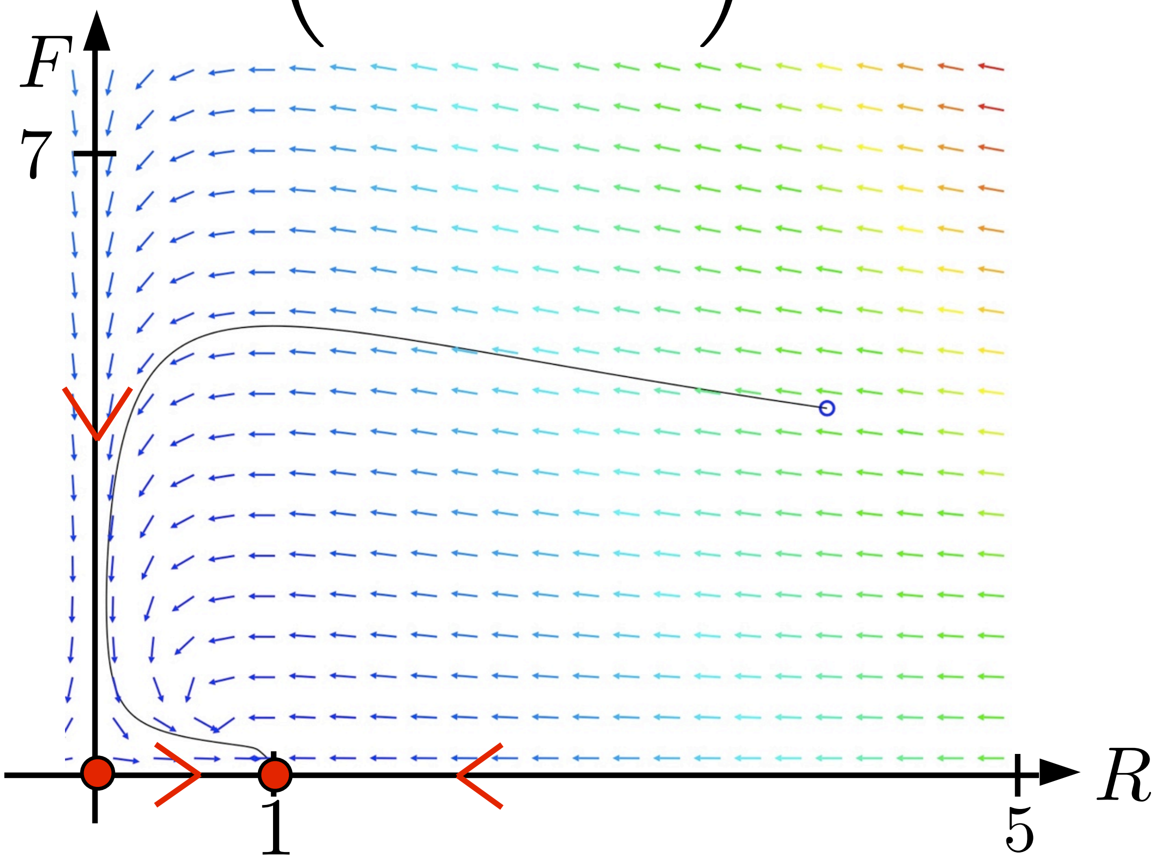
$$\frac{dF}{dt} = F \cdot \Big(-1 + R \Big)$$



- rabbit vs foxes: example 3

$$\frac{dR}{dt} = 2R \cdot \left(1 - R\right) - R \cdot F$$

$$\frac{dF}{dt} = F \cdot \left(-1 + R\right)$$



Linear Systems

$$\begin{cases} \dfrac{dx}{dt} = 2x - y \\ \dfrac{dy}{dt} = 3x - 2y \end{cases}$$

- We are going to learn how to find all solutions.
- What if this Joe Schmoe gives you a solution:

$$\vec{\mathbf{Y}}(t) = \begin{pmatrix} e^t \\ e^t \end{pmatrix} \quad \vec{\mathbf{Y}}(t) = \begin{pmatrix} e^{2t} \\ e^t \end{pmatrix}$$

CHECK IT

General Solution

$$\frac{d\vec{\mathbf{Y}}}{dt} = F(\vec{\mathbf{Y}}, t)$$

- General Solution: Family of solutions from which we can solve any initial value problem.
- initial value problem: $\vec{\mathbf{Y}}(0) =$ any vector.

General Solution-Examples

- Simple Ex:

$$\frac{dx}{dt} = x$$

$$\frac{dy}{dt} = -y$$

one solution	general solution
$x(t) = e^t$	$x(t) = k_1 e^t$?
$y(t) = e^{-t}$	$y(t) = k_2 e^{-t}$

- This is the general solution if we can find k_1, k_2 so that $x = k_1 e^t$, $y = k_2 e^{-t}$ solves the initial value problem

$$\vec{\mathbf{Y}}(0) = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \text{ where } \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \text{ is any vector}$$

Linearity Principle

- Linear system of differential equations

$$\frac{d\vec{\mathbf{Y}}}{dt} = \mathbf{A} \cdot \vec{\mathbf{Y}}$$

- $\mathbf{A}$ nxn matrix and $\vec{\mathbf{Y}}$ nx1 vector.

- if $\vec{\mathbf{Y}}(t)$ is a of the system and k is any constant the k$\vec{\mathbf{Y}}(t)$ is also a solution.

- if $\vec{\mathbf{Y}}_1(t)$ and $\vec{\mathbf{Y}}_2(t)$ are two solutions of this system, then $\vec{\mathbf{Y}}_1(t) + \vec{\mathbf{Y}}_2(t)$ is also a solution.

Principle of Superposition

- if $\vec{\mathbf{Y}}_1(t)$ and $\vec{\mathbf{Y}}_2(t)$ are two solutions

$$k_1\vec{\mathbf{Y}}_1(t) + k_2\vec{\mathbf{Y}}_2(t)$$

  also a solutions

- called linear combination

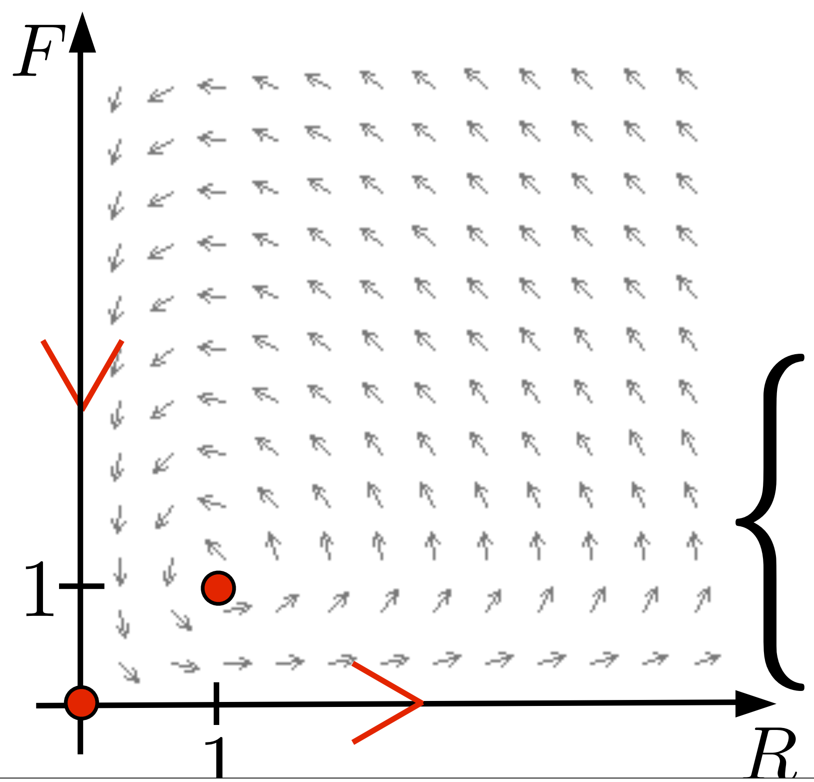
Linear Independence

- Two vectors are linearly independent if the do not lie on the same line through the origin.

- Equivalently if neither one is a multiple of the other

Predator-Prey Systems

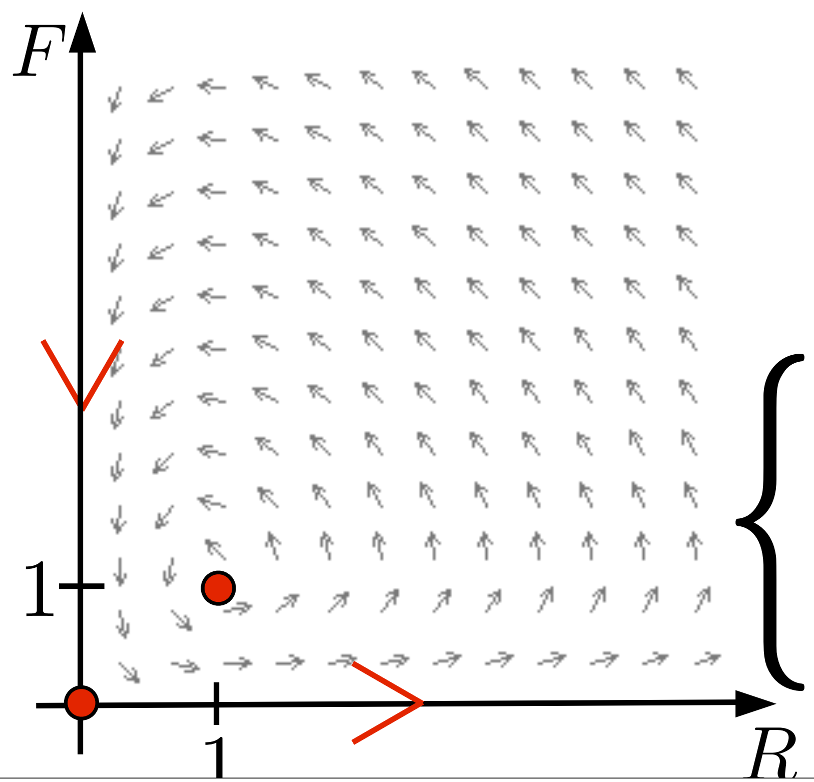
- Assumptions
  - phase plane (F vs R)



can't get further analytically

Predator-Prey Systems

- Assumptions
  - phase plane (F vs R)



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